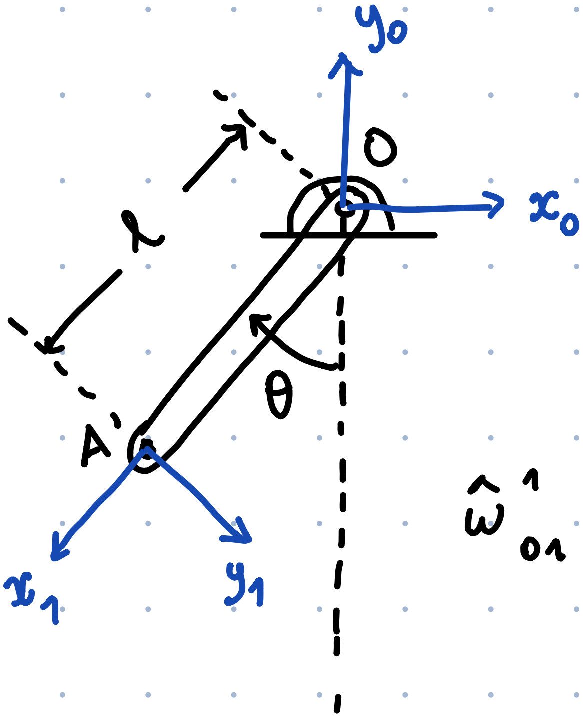


$$R_{01} = \begin{bmatrix} s_\theta & c_\theta & 0 \\ -c_\theta & s_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\omega}^1_{01} = R_{01}^T \dot{R}_{01}$$

$$= \begin{bmatrix} s_\theta & -c_\theta & 0 \\ c_\theta & s_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

$$\Rightarrow \quad {}^0\omega^1 = \dot{\theta} z_1$$

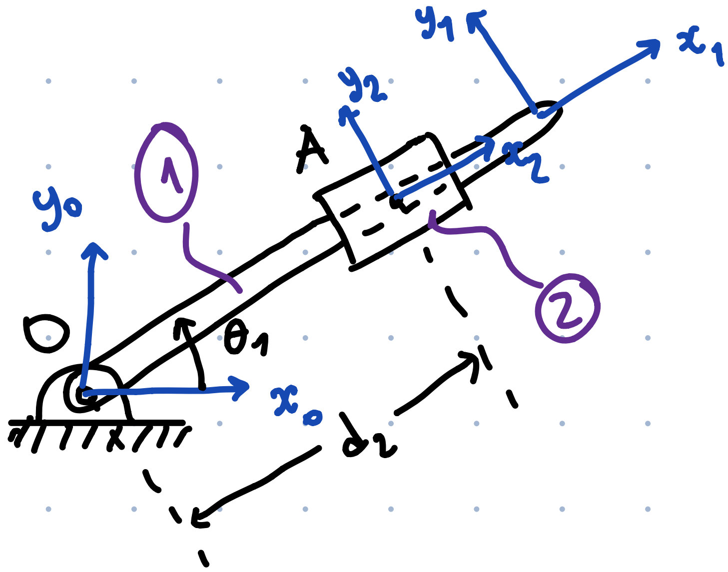
Let p be the position vector from O to A.
Then, $p = l x_1$. We compute:

$$\frac{{}^0 d}{dt} p = {}^0\omega^1 \times p = \dot{\theta} z_1 \times l x_1 = l \dot{\theta} y_1$$

match!

Or, $p = l x_1 = l s_\theta x_0 - l c_\theta y_0$, so,

$$\frac{{}^0 d}{dt} p = \frac{d}{dt}(l s_\theta) x_0 - \frac{d}{dt}(l c_\theta) y_0 = l \dot{\theta}(c_\theta x_0 + s_\theta y_0) = l \dot{\theta} y_1$$

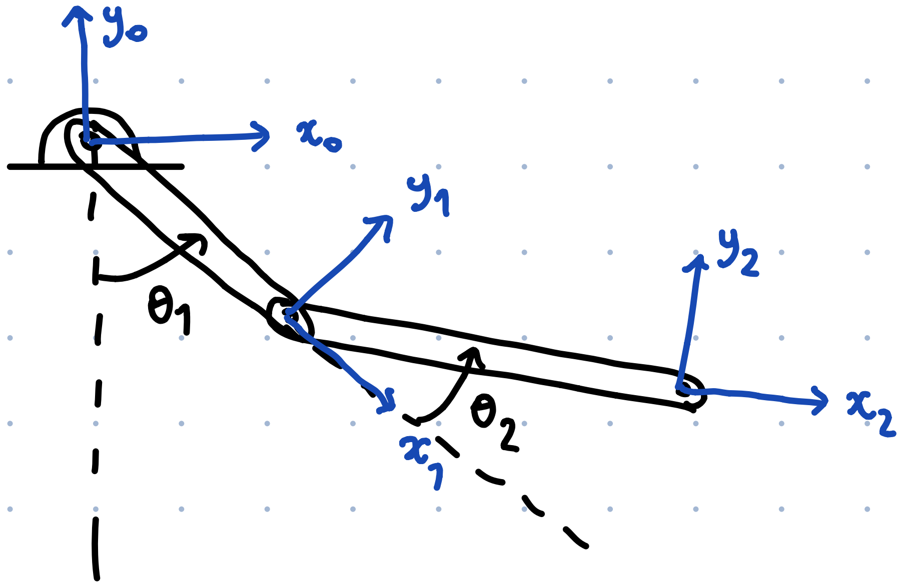


$$p = \overrightarrow{OA} = d_2 x_2$$

$$^0\omega^2 = {}^0\omega^1 = \dot{\theta}_1 z_1$$
$$= \dot{\theta}_1 z_2$$

$$\frac{{}^0 dp}{dt} = \frac{{}^2 dp}{dt} + {}^0\omega^2 \times p \quad = \frac{{}^2 d}{dt}\left(d_2 x_2\right) + \dot{\theta}_1 z_2 \times d_2 x_2$$

$$= \dot{d}_2 x_2 + \dot{\theta}_1 z_2 \times d_2 x_2 = \dot{d}_2 x_2 + d_2 \dot{\theta}_1 y_2 \,.$$



$$R_{01} = \begin{bmatrix} s_1 & c_1 & 0 \\ -c_1 & s_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow {}^0\hat{\omega}^1 = R_{01}^T \dot{R}_{01} = \begin{bmatrix} s_1 & -c_1 & 0 \\ c_1 & s_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}_1 = \dot{\theta}_1 \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow {}^0\omega^1 = \dot{\theta}_1 z_1$$

$$R_{12} = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^1\hat{\omega}^2 = R_{12}^T \dot{R}_{12} = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s_2 & -c_2 & 0 \\ c_2 & -s_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}_2$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}_2$$

$$\Rightarrow {}^1\omega^2 = \dot{\theta}_2 z_2$$

Addition Thm. of Angular Velocities:

$${}^0\omega^2 = {}^0\omega^1 + {}^1\omega^2 = \dot{\theta}_1 z_1 + \dot{\theta}_2 z_2 = \dot{\theta}_1 z_2 + \dot{\theta}_2 z_2 = (\dot{\theta}_1 + \dot{\theta}_2) z_2.$$

On the other hand,

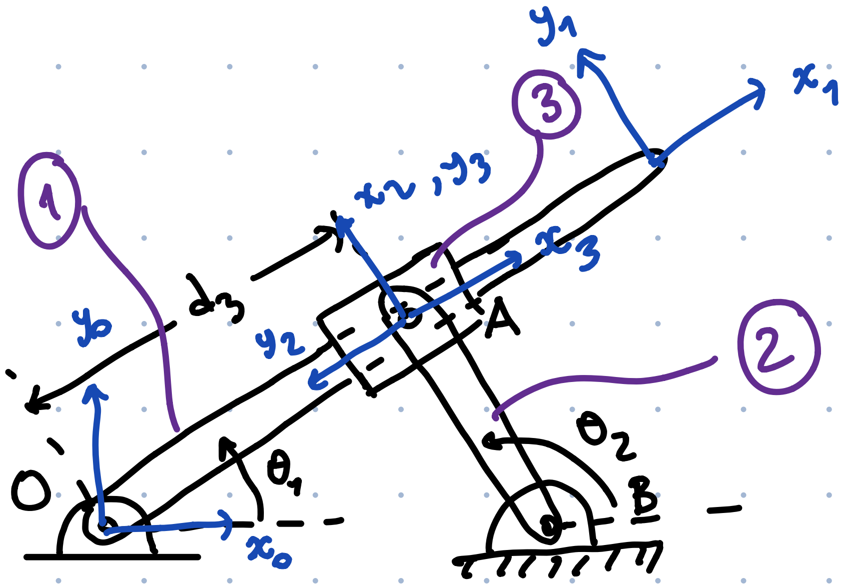
$$R_{02} = R_{01} R_{12} = \begin{bmatrix} s_1 & c_1 & 0 \\ -c_1 & s_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1 c_2 + c_1 s_2 & c_1 c_2 - s_1 s_2 & 0 \\ -(c_1 c_2 - s_1 s_2) & s_1 c_2 + c_1 s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{12} & c_{12} & 0 \\ -c_{12} & s_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^0\hat{\omega}^2 = R_{02}^T \dot{R}_{02} = \begin{bmatrix} s_{12} & -c_{12} & 0 \\ c_{12} & s_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} (\dot{\theta}_1 + \dot{\theta}_2)$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (\dot{\theta}_1 + \dot{\theta}_2)$$

$$\Rightarrow \; ^0\omega^2 = (\dot{\theta}_1 + \dot{\theta}_2) z_2$$



Let $p = \overrightarrow{OA} = \overrightarrow{OA_1} + \overrightarrow{A_1A_3}$ (A_1 is on body 1, A_3 is on body 3)

$= p_1 + p_2$

$$\frac{{}^{0}d}{dt} p = \frac{{}^{0}d}{dt}(p_1 + p_2) = \frac{{}^{1}d}{dt} p_1 + {}^{0}\omega^{1} \times p_1 + \frac{{}^{3}d}{dt} p_2 + {}^{0}\omega^{3} \times p_3$$

$$= {}^{0}\omega^{1} \times p_1 + {}^{1}v^{A_3}$$

$$\frac{{}^{0}d}{dt} p = \frac{{}^{3}d}{dt} p + {}^{0}\omega^{3} \times p = -\left(\frac{{}^{3}d}{dt}(-p)\right) + {}^{0}\omega^{3} \times p$$

$$= -{}^{3}v^{0} + {}^{0}\omega^{3} \times p$$