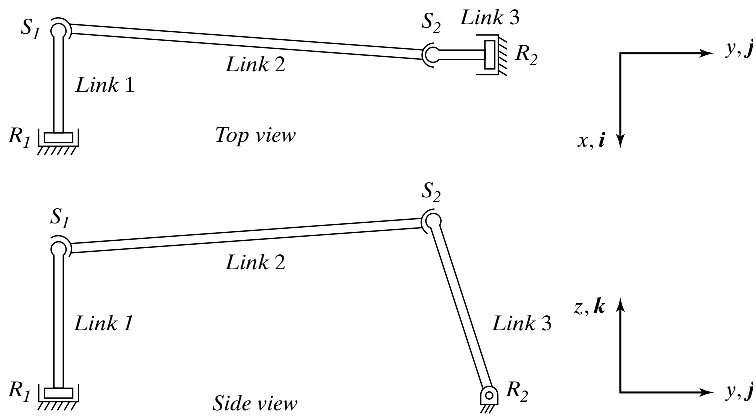


FIGURE P3.1 *RSSR* spatial linkage.

(all in millimeters). Link 1 rotates at $\omega_1 = 25$ rad/s (constant) in the xz-plane. Sphere joint S_1 is moving away from the observer at this instant. Link 3 rotates in the yz-plane. Find velocities v_{s_1} and v_{s_2} and angular velocity ω_3. Set the angular velocity of link 2 about its own axis to 0.

3.25 Repeat Problem 3.24, except that $\omega_3 = 10$ rad/s cw (constant). Find $\mathbf{v}_{s_1}$, $\mathbf{v}_{s_2}$, and ω_1.

3.26 **(a)** Find the crank position corresponding to the maximum piston velocity for an in-line slider-crank mechanism. Crank speed ω is constant, and the ratio of the connecting rod to the crank length is $L/R = 2$.

(b) Find the maximum piston velocity in terms of R and ω.

3.27 Repeat Problem 3.26 for $L/R = 1.5$.

In Problems 3.28 through 3.61,

(a) *write the appropriate vector equation*,

(b) *solve the equation graphically unless directed otherwise, using velocity polygon notation*,

(c) *dimension all vectors of the velocity polygon*, and

(d) *express angular velocities in radians per second, and indicate their directions*.

3.28 In Figure P3.2, $\theta = 45°$ and $\omega_1 = 10$ rad/s. Draw and dimension the velocity polygon. Find ω_2.

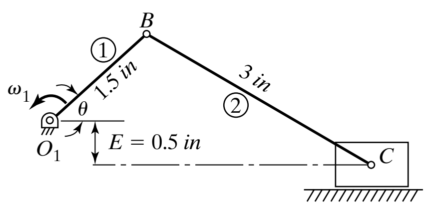


FIGURE P3.2

3.29 Repeat Problem 3.28 for $\theta = 120°$.

3.30 Repeat Problem 3.28 for the mechanism in the limiting position (with C to the extreme right).

3.31 In Figure P3.3, $\omega_1 = 100$ rad/s. Draw and dimension the velocity polygon. Find v_D and ω_2. Use the scale 1 in = 100 in/s.

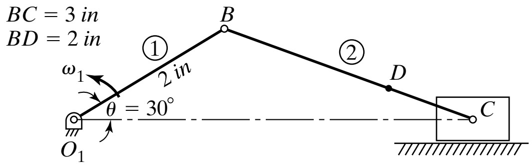


FIGURE P3.3

3.32 In Figure P3.4, $\omega_1 = 50$ rad/s. Draw and dimension the velocity polygon. Find v_D, ω_2, and ω_3. Use the scale 1 in = 50 in/s.

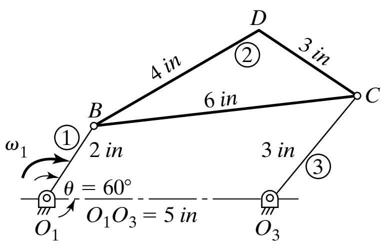


FIGURE P3.4

3.33 In Figure P3.5, $\omega_1 = 20$ rad/s.

(a) Draw and dimension the velocity polygon for the limiting position shown. Find relative velocity v_{CB}. Use the scale 1 in = 10 in/s.

(b) Repeat the problem for the other limiting position.

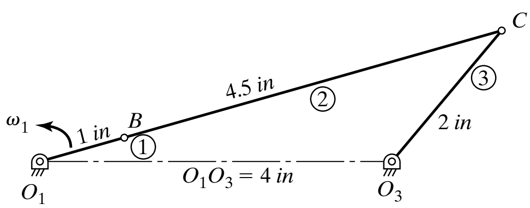


FIGURE P3.5

3.34 In Figure P3.6, $\theta = 105°$ and $\omega_2 = 20$ rad/s. Draw and dimension the velocity polygon. Identify the sliding velocity. Find ω_1. Use the scale 1 in = 20 in/s.

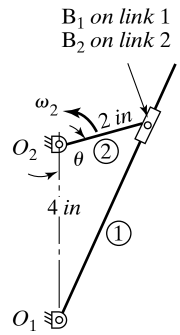


FIGURE P3.6

3.35 Repeat Problem 3.34 for $\theta = 30°$.

3.36 In Figure P3.3, $\omega_1 = 100$ rad/s.

(a) Find $\boldsymbol{v}_C$ *analytically* for the position shown.

(b) Find $\boldsymbol{v}_C$ analytically at both limiting positions.

3.37 In Figure P3.7, let the angular velocity of the crank be ω.

(a) Draw the velocity polygon for the position shown. Identify relative velocity v_{CB}.

(b) Repeat for the other limiting position.

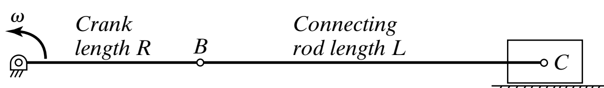


FIGURE P3.7

3.38 In Figure P3.8, $\omega_1 = 100$ rad/s. Draw and dimension the velocity polygon. Use the scale 1 in = 100 in/s.

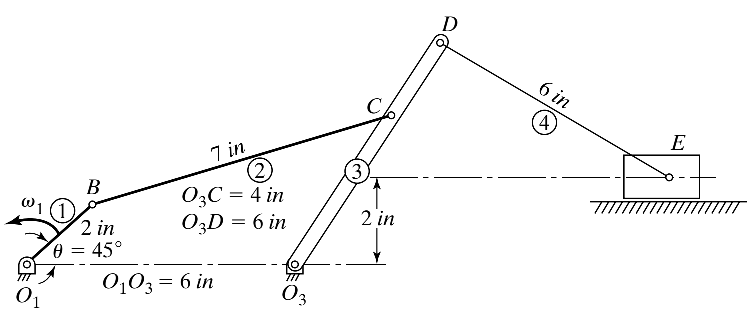


FIGURE P3.8

3.39 In Figure P3.9, $\theta = 135°$ and $\omega_1 = 10$ rad/s. Draw and dimension the velocity polygon. Identify the follower velocity and the sliding velocity. Use the scale 1 in = 5 in/s.

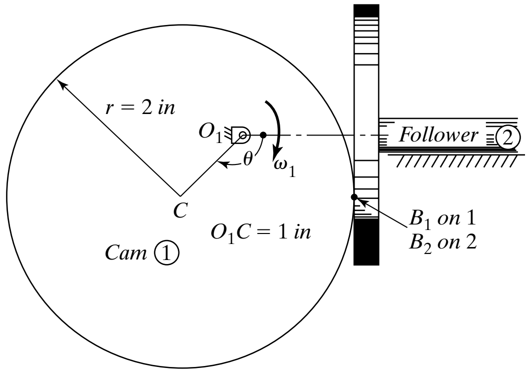


FIGURE P3.9

3.40 Repeat Problem 3.39 for $\theta = 30°$.

3.41 In Figure P3.10, $\omega_1 = 35$ rad/s. Draw the velocity polygon. Use the scale 1 in = 10 in/s. Find ω_2 and ω_3. Find the velocity of the midpoint of each link.

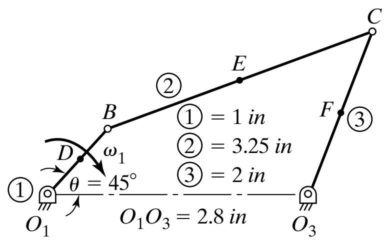


FIGURE P3.10

3.42 In Figure P3.11, $\omega_1 = 20$ rad/s.

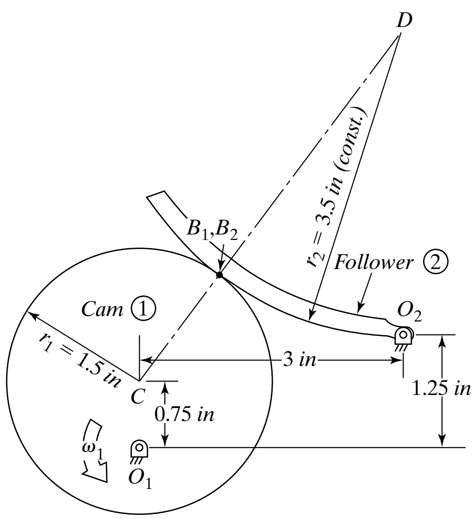


FIGURE P3.11

(a) Draw and dimension the velocity polygon. Find ω_2 and identify the sliding velocity.

(b) Note that CD is a fixed distance. Thus, we can use the equivalent linkage O_1CDO_2. Draw the velocity polygon, and again, find the angular velocity of the follower (represented by DO_2).

3.43 Consider a pair of involute spur gears with a 20° pressure angle. Let the driver speed be 300 rev/min clockwise and the driven gear speed 1,000 rev/min counterclockwise. Find the sliding velocity when contact occurs 1.2 in from the pitch point.

3.44 In Figure P3.12, $\theta = 105°$ and $\omega_2 = 20$ rad/s. Draw and dimension the velocity polygon, using the scale 1 in $=$ 20 in/s.

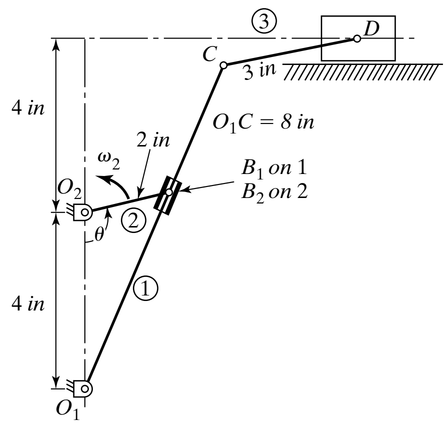


FIGURE P3.12

3.45 Repeat Problem 3.44 for $\theta = 30°$.

3.46 In Figure P3.9, $\omega_1 = 10$ rad/s. Find the follower velocity analytically when **(a)** $\theta = 135°$ and **(b)** $\theta = 30°$.

3.47 In Figure P3.13, ω_1(the angular velocity of O_1B_1) $= 30$ rad/s. Draw and dimension the velocity polygon, using the scale 1 in $=$ 20 in/s. Identify the sliding velocity and find ω_2, the angular velocity of O_2B_2.

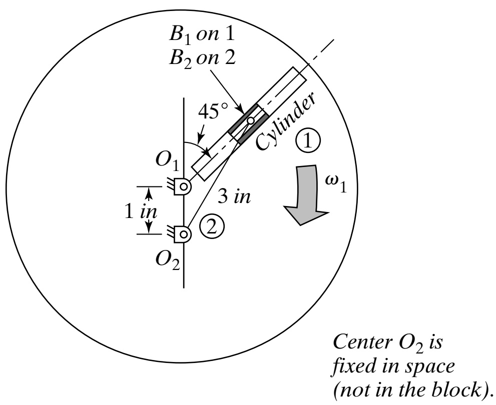


FIGURE P3.13

3.48 For Figure P3.14, draw and dimension the velocity polygon, using the scale 1 in $=$ 100 in/s. Find ω_2 and v_D. Points B, C, and D lie on the same rigid link.

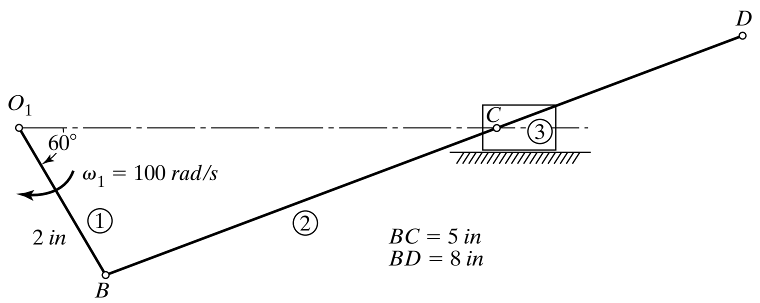


FIGURE P3.14

3.49 Locate all of the centros in Figure P3.14. Using centro 13, write an expression for the slider velocity in terms of ω_1. Calculate $\boldsymbol{v}_C$.

3.50 In Figure P3.14, use centro 02 in order to write an expression for **(a)** ω_2 in terms of v_B; **(b)** ω_2/ω_1; **(c)** v_C in terms of ω_2; and **(d)** v_D in terms of ω_2. **(e)** Calculate ω_2, v_C, and v_D.

3.51 In Figure P3.15, $\omega_5 = 15$ rad/s. Use a vector 3 in long to represent the velocity of B. Complete the velocity polygon and determine the velocity scale. Dimension the polygon and find the angular velocity of each link.

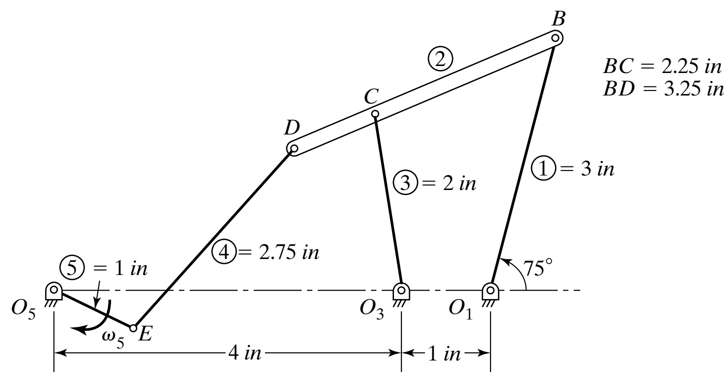


FIGURE P3.15

3.52 Refer to Figure P3.16. Solve graphically for $\theta_1 = 15°$.

(a) Draw the linkage to a 1:1 scale.

(b) Let 1 mm $=$ 5 mm/s; draw velocity polygon $\boldsymbol{ob_1b_2}$. Add point c, where $O_2C = 100$ m.

(c) Find v_{B2}.

(d) Find v_c.

(e) Find ω_2.

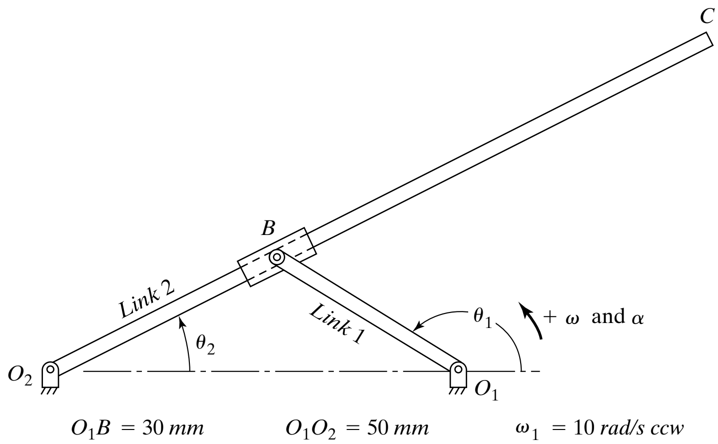


FIGURE P3.16

3.53 Repeat Problem 3.52 for $\theta_1 = 30°$.

3.54 Repeat Problem 3.52 for $\theta_1 = 45°$.

3.55 Refer to Figure P3.16. Find θ_1 when $\omega_2 = 0$.

3.56 Refer to Figure P3.17.

(a) Draw velocity polygon ***obc***.

(b) Find ω_2.

(c) Locate d on the velocity polygon. Find v_D.

(d) Identify angles θ and ϕ in your solution. Find velocity v_c and relative velocity bc in terms of ω_1, O_1B, θ, and ϕ.

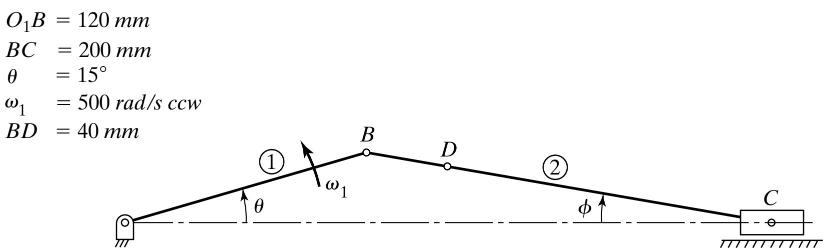


FIGURE P3.17

3.57 A four-bar planar linkage has the following dimensions:

$$\begin{aligned} r_0\,(\text{frame}) &= 4; \\ r_1\,(\text{driver}) &= 1.25; \\ r_2\,(\text{coupler}) &= 5.25; \\ r_3\,(\text{driven link}) &= 2.5. \end{aligned}$$

If $\omega_1 = 5.75$ rad/s cw (constant), plot ω_2 and ω_3 versus θ. Use a computer or programmable calculator. Check values at $\theta = 120°$ by using a velocity polygon.

3.58 In Figure P3.18, $\theta_2 = 135°$, $\omega_2 = 500$ rad/s ccw (constant), $O_1O_2 = 300$ mm, $O_1C = 400$ mm, $O_2B = 150$ mm, and $CD = 160$ mm.

(a) Draw velocity polygon $\boldsymbol{ob_1b_2}$.

(b) Find $\boldsymbol{oc}$.

(c) Find ω_1.

(d) Add $\boldsymbol{c}$ to the polygon.

(e) Add $\boldsymbol{d}$ to the polygon.

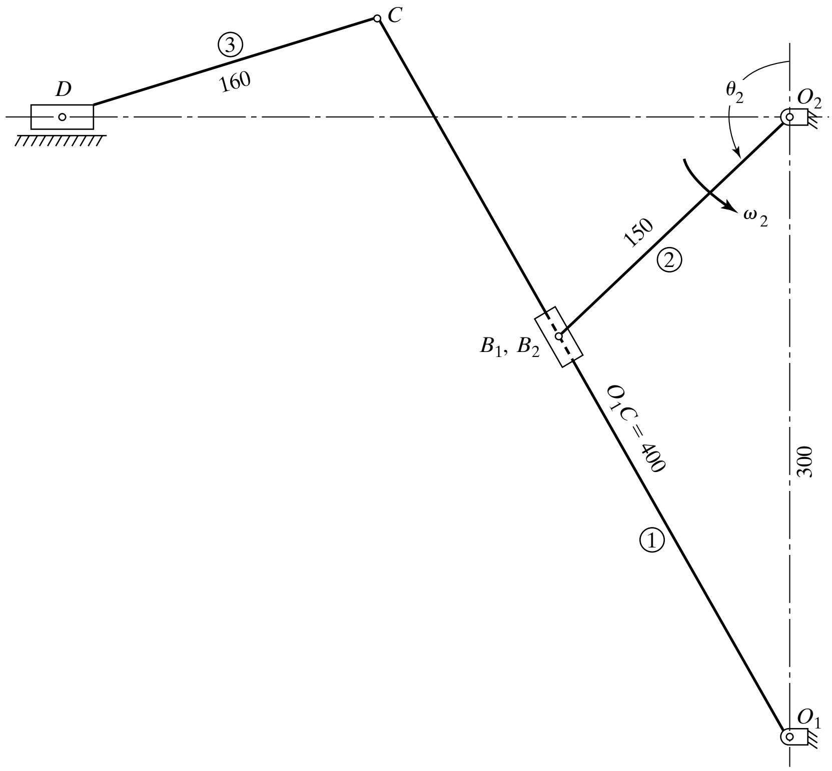


FIGURE P3.18

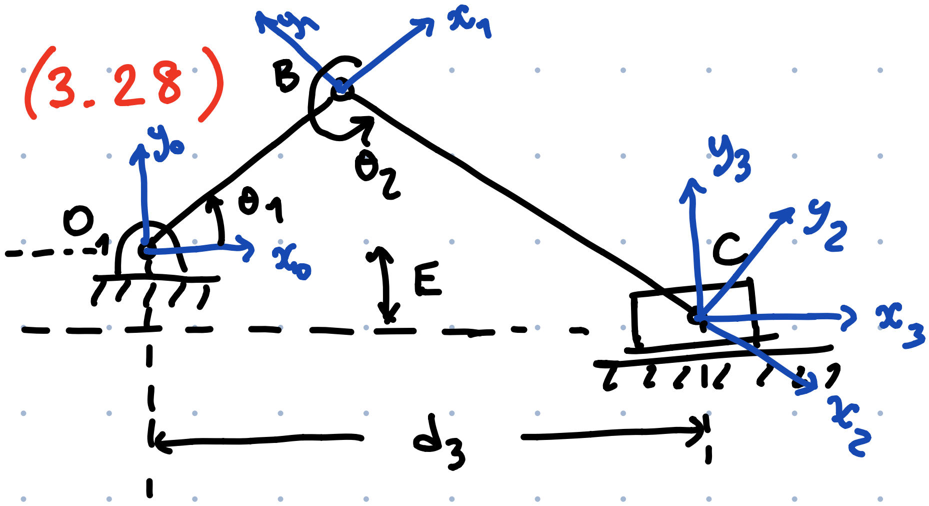
3.59 In Problem 3.58, let $\theta_2 = 120°$.

(a) Find v_{B_1}, v_D, and ω_1 at $\theta_2 = 120°$.

(b) For the interval $120° < \theta_2 < 135°$, find the average angular acceleration of link 1 and the average acceleration of point D.

3.60 Refer to Figure 3.15. Let $R_1 = 30$ and $O_1O_2 = 50$. Find θ_1 when $\omega_2 = 0$.

(3.28)



$\theta_1 = \frac{\pi}{4}$ $\quad l_1 = 1.5$ in

$l_2 = 3$ in.

${}^0\omega^1 = 10\, z_1$ $\quad E = \frac{1}{2}$ in.

${}^0\omega^2 = ?$

Position-level analysis:

$$R_{01} = \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_{12} = \begin{pmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{02} = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Loop eqn.: $\overbrace{l_1 x_1}^{p_1} + \overbrace{l_2 x_2}^{p_2} \overbrace{- d_3 x_0 + E y_0}^{p_0} = 0$

$$\begin{bmatrix} l_1 c_1 \\ l_1 s_1 \end{bmatrix} + \begin{bmatrix} l_2 c_{12} \\ l_2 s_{12} \end{bmatrix} - \begin{bmatrix} d_3 \\ E \end{bmatrix} = 0$$

Inserting values and solving yields:

$\theta_2 = -0.9734$ rad $= -55.77°$

$d_3 = 4.008$ in.

Velocity-level analysis

Differentiate the loop equation w.r.t. time in the world frame:

$${}^0\frac{d}{dt}\left(p_0 + p_1 + p_2\right) = 0$$

$$\Rightarrow {}^0\frac{d}{dt} p_0 + {}^0\frac{d}{dt} p_1 + {}^0\frac{d}{dt} p_2 = 0$$

Use differentiation in two frames formula to get:

$$-\dot{d}_3 x_0 + {}^0\omega^1 \times p_1 + {}^0\omega^2 \times p_2 = 0,$$

$$\Rightarrow -\dot{d}_3 x_0 + \dot{\theta}_1 z_1 \times l_1 x_1 + \dot{\theta}_2 z_2 \times l_2 x_2 = 0,$$

$$\Rightarrow -\dot{d}_3 x_0 + l_1 \dot{\theta}_1 y_1 + l_2 \dot{\theta}_2 y_2 = 0, \qquad (*)$$

where

$${}^0\hat{\omega}^1 = R_{01}^T \dot{R}_{01} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}_1 \quad \text{and}$$

$${}^0\hat{\omega}^2 = R_{02}^T \dot{R}_{02} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}_2$$

Expressing every vector in $(*)$ in Σ_0, we get

$$\begin{bmatrix} -\dot{d}_3 - l_1 s_1 \dot{\theta}_1 - l_2 s_2 \dot{\theta}_2 \\ l_1 c_1 \dot{\theta}_1 + l_2 c_2 \dot{\theta}_2 \end{bmatrix}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^0$$

Rearranging this equation, we get

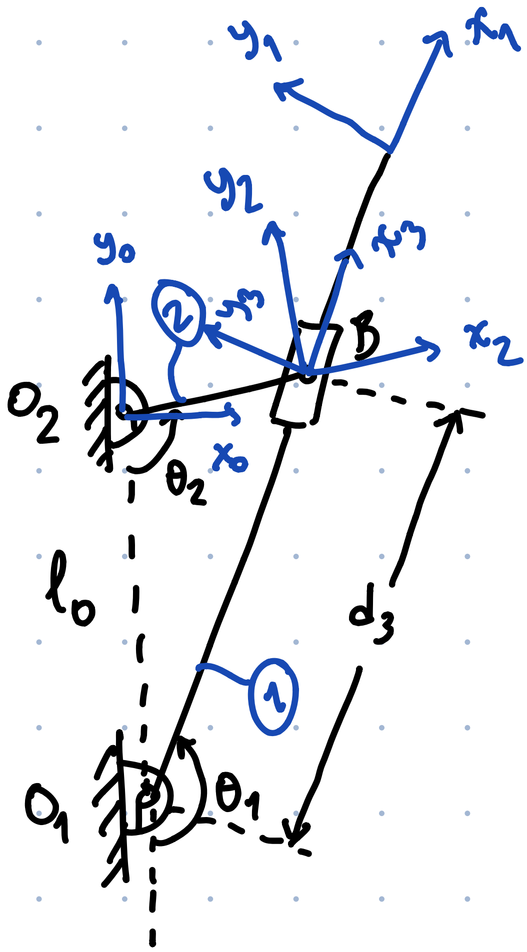
$$\begin{bmatrix} -l_2 \sin(\theta_2) & -1 \\ l_2 \cos(\theta_2) & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} l_1 \sin(\theta_1) \\ -l_1 \cos(\theta_1) \end{bmatrix} \dot{\theta}_1$$

Now, plugging in the values for $l_1, l_2, \theta_1, \theta_2, d_3, \dot{\theta}_1$ we get

$$\begin{bmatrix} 2.480 & -1 \\ 1.687 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} 10.61 \\ -10.61 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -6.285 \\ -26.197 \end{bmatrix} \begin{matrix} \rightarrow \text{rad/s} \\ \rightarrow \text{in/s} \end{matrix}$$

(3.34)



$$\theta_2 = \frac{7\pi}{12}$$

$$^0\omega^2 = \dot{\theta}_2 z_2 = 20 z_2 \ \frac{\text{rad}}{\text{s}}.$$

$$l_0 = 4 \text{ in}$$
$$l_2 = 2 \text{ in.}$$

Position-level analysis.

$$p_0 = \overrightarrow{O_1O_2}, \quad p_2 = \overrightarrow{O_2B}, \quad p_1 = \overrightarrow{O_1B}$$

$$p_0 - p_1 + p_2 = 0. \quad (*)$$

$$R_{02} = \begin{bmatrix} s_2 & c_2 & 0 \\ -c_2 & s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad R_{01} = \begin{bmatrix} s_1 & c_1 & 0 \\ -c_1 & s_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ l_0 \\ 0 \end{bmatrix}^0 - \begin{bmatrix} d_3 s_1 \\ -d_3 c_1 \\ 0 \end{bmatrix}^0 + \begin{bmatrix} l_2 s_2 \\ -l_2 c_2 \\ 0 \end{bmatrix}^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^0$$

$$\Rightarrow \begin{aligned} l_2 s_2 - d_3 s_1 &= 0 \\ l_0 - l_2 c_2 + d_3 c_1 &= 0 \end{aligned}$$

Plugging in the givens and solving for θ_1 and d_3 yields

$$\theta_1 = 2.7375 \text{ rad} = 156.85^\circ$$
$$d_3 = 4.9133 \text{ in.}$$

Velocity-level analysis

$$ {}^{0}\hat{\omega}^{1} = R_{01}^{T}\dot{R}_{01} = \begin{pmatrix} s_1 & -c_1 & 0 \\ c_1 & s_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\dot{\theta}_1 $$

$$ = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\dot{\theta}_1 \Rightarrow {}^{0}\omega^{1} = \dot{\theta}_1 z_1 = {}^{0}\omega^{3} $$

Similarly ${}^{0}\omega^{2} = \dot{\theta}_2 z_2 = 20 z_2$ (given)

Differentiate the loop equation, (*), in frame Σ_0:

$$ \frac{{}^{0}d}{dt}\left(p_0 - p_1 + p_2\right) = 0 $$

$$ \Rightarrow \frac{{}^{0}d}{dt}p_1 - \frac{{}^{0}d}{dt}p_2 = 0 \qquad (p_0 \text{ is fixed in } \Sigma_0). $$

$$ \Rightarrow \frac{{}^{3}d}{dt}p_1 + {}^{0}\omega^{3} \times p_1 - \frac{{}^{2}d}{dt}p_2 - {}^{0}\omega^{2} \times p_2 = 0 $$
(diff. in two frames)

$$ \Rightarrow \frac{{}^{3}d}{dt}\left(d_3 x_3\right) + \dot{\theta}_1 z_3 \times d_3 x_3 - \dot{\theta}_2 z_2 \times l_2 x_2 = 0 $$
(plug variables in)

$$ \Rightarrow \dot{d}_3 x_3 + d_3 \dot{\theta}_1 y_3 - l_2 \dot{\theta}_2 y_2 = 0 $$
(perform differentiation and cross product).

We express this equation in Σ_0:

$$\begin{bmatrix} \dot{d}_3 s_1 \\ -\dot{d}_3 c_1 \end{bmatrix}^0 + \begin{bmatrix} d_3 c_1 \dot{\theta}_1 \\ d_3 s_1 \dot{\theta}_1 \end{bmatrix}^0 - \begin{bmatrix} l_2 c_2 \dot{\theta}_2 \\ l_2 s_2 \dot{\theta}_2 \end{bmatrix}^0 = 0$$

Rearranging this equation yields:

$$\begin{bmatrix} d_3 \cos(\theta_1) & \sin(\theta_1) \\ d_3 \sin(\theta_1) & -\cos(\theta_1) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} l_2 \cos(\theta_2) \\ l_2 \sin(\theta_2) \end{bmatrix} \dot{\theta}_2$$

Plugging in the knowns: $d_3, \theta_1, \theta_2, l_2, \dot{\theta}_2$ yields

$$\begin{bmatrix} -4.518 & 0.3932 \\ 1.9318 & 0.9195 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -10.353 \\ 38.637 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} 5.029 \\ 31.455 \end{bmatrix} \begin{matrix} \rightarrow \text{rad/s} \\ \rightarrow \text{in/s} \end{matrix} .$$