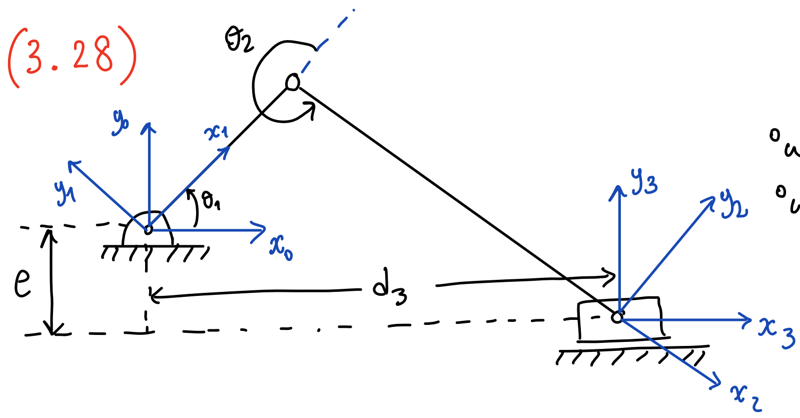
(3.28)



$\theta_1 = \pi/4$ $\quad$ $\ell_1 = 1.5$

${}^0\omega^1 = 10\, z_1$ $\quad$ $\ell_2 = 3$

${}^0\omega^2 = ?$ $\quad$ $e = 1/2$

Position-level analysis

$$R_{01} = R_{z,\theta_1} \quad , \quad R_{12} = R_{z,\theta_2} \quad ; \quad R_{02} = R_{01}R_{12} = R_{z,\theta_1+\theta_2}$$

Loop equation: $\underbrace{\ell_1 x_1}_{P_1} + \underbrace{\ell_2 x_2}_{P_2} \underbrace{- d_3 x_0 + e y_0}_{P_0} = 0$

In Σ_0 :
$$\ell_1 \begin{bmatrix} c_1 \\ s_1 \end{bmatrix} + \ell_2 \begin{bmatrix} c_{12} \\ s_{12} \end{bmatrix} + \begin{bmatrix} -d_3 \\ e \end{bmatrix} = 0$$

Solving these equations for θ_2 and d_3 given $\theta_1 = \pi/4$, we get

$$\theta_2 = 4.950679 \text{ rad} = 283.653^\circ$$

$$d_3 = 3.622757 .$$

Velocity-level analysis

$$S({}^1\omega^2) = R_{12}^T \dot{R}_{12} = \dot{\theta} \begin{pmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -s_2 & -c_2 & 0 \\ c_2 & -s_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \dot{\theta}_2 \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \; {}^1\omega^2 = \dot{\theta}_2 \, z_2$$

By the addition of angular velocities, and the fact that $z_1 = z_2$, we have

$$^0\omega^2 = {}^0\omega^1 + {}^1\omega^2 = \dot{\theta}_1 z_1 + \dot{\theta}_2 z_2 = (\dot{\theta}_1 + \dot{\theta}_2) z_2$$

Now, let us differentiate w.r.t. time the position-level loop equation:

$$\frac{{}^0 d}{dt}\left[p_0 + p_1 + p_2 = 0 \right] \Rightarrow \frac{{}^0 d}{dt} p_0 + \frac{{}^0 d}{dt} p_1 + \frac{{}^0 d}{dt} p_2 = 0$$

$$\Rightarrow -\dot{d}_3 x_0 + \underbrace{{}^0\omega^1 \times p_1}_{\substack{\dot{\theta}_1 z_1 \times l_1 x_1 \\ \parallel \\ l_1 \dot{\theta}_1 y_1}} + \underbrace{{}^0\omega^2 \times p_2}_{\substack{(\dot{\theta}_1 + \dot{\theta}_2) z_2 \times l_2 x_2 \\ \parallel \\ l_2 (\dot{\theta}_1 + \dot{\theta}_2) y_2}} = 0$$

$$\Rightarrow -\dot{d}_3 x_0 + l_1 \dot{\theta}_1 y_1 + l_2 (\dot{\theta}_1 + \dot{\theta}_2) y_2 = 0.$$

In Σ_0, we have

$$\begin{bmatrix} -\dot{d}_3 \\ 0 \end{bmatrix} + l_1 \dot{\theta}_1 \begin{bmatrix} -s_1 \\ c_1 \end{bmatrix} + l_2 (\dot{\theta}_1 + \dot{\theta}_2) \begin{bmatrix} -s_{12} \\ c_{12} \end{bmatrix} = 0.$$

In matrix form, we obtain

$$\underbrace{\begin{bmatrix} -l_2 s_{12} & -1 \\ l_2 c_{12} & 0 \end{bmatrix}}_{\begin{bmatrix} 1.56066 & -1 \\ 2.5621 & 0 \end{bmatrix}} \begin{bmatrix} \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \underbrace{\left(l_1 \begin{bmatrix} s_1 \\ -c_1 \end{bmatrix} + l_2 \begin{bmatrix} s_{12} \\ -c_{12} \end{bmatrix} \right) \dot{\theta}_1}_{\begin{bmatrix} -5 \\ -36.227569 \end{bmatrix}}$$

$= 10 \text{ rad/s}$

Plugging in the known values and solving yields

$$\begin{bmatrix} \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -14.139813 \\ -17.067443 \end{bmatrix}$$

Hence,

$$^0\omega^2 = {}^0\omega^1 + {}^1\omega^2 = (\dot{\theta}_1 + \dot{\theta}_2) z_2$$

$$= (10 - 14.139813) z_2 = -4.139813 z_2 .$$