ME 380: Kinematics and Machine Dynamics
Fall 2021 | Midterm Examination #2

Name: ______________________________

Question:	1	2	3	Total
Points:	40	60	0	100
Bonus Points:	0	0	25	25
Score:				

Angle sum and difference identities

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

Isomorphism between $\mathfrak{so}(3)$ and $\mathbb{R}^3$

Let $a = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}^\top \in \mathbb{R}^3$ be a vector. There is a unique correspondence between a 3×3 skew-skymmetric matrix, $S(a)$ and the vector a:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = a \leftrightarrow S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

Angular and linear velocities

We define the (body) angular velocity and angular acceleration of a reference frame Σ_B with respect to Σ_A, respectively, by

$$S\left({}^A\omega^B\right) = R_{ab}^\top \dot{R}_{ab}, \qquad {}^A\alpha^B = \frac{{}^A\mathrm{d}\,{}^A\omega^B}{\mathrm{d}t}.$$

Suppose that $r \in \mathbb{R}^3$ is a vector that begins at *any* point fixed in frame Σ_A and ends at a point, Q, moving in Σ_A. We define the linear velocity and acceleration of point Q in Σ_A, respectively, by

$${}^Av^Q \triangleq \frac{{}^A\mathrm{d}r}{\mathrm{d}t}, \qquad {}^Aa^Q \triangleq \frac{{}^A\mathrm{d}\,{}^Av^Q}{\mathrm{d}t}.$$

Differentiation in two frames

If Σ_A and Σ_B are two reference frames, the first time-derivative of *any* vector v in Σ_A and in Σ_B are related to each other as follows:

$$\frac{{}^A\mathrm{d}v}{\mathrm{d}t} = \frac{{}^B\mathrm{d}v}{\mathrm{d}t} + {}^A\omega^B \times v.$$

Auxiliary reference frames

The angular velocity of a rigid body B in a reference frame A can be expressed in the following form involving n auxiliary reference frames $A_1, \ldots, A_n$:

$${}^A\omega^B = {}^A\omega^{A_1} + {}^{A_1}\omega^{A_2} + \cdots + {}^{A_{n-1}}\omega^{A_n} + {}^{A_n}\omega^B.$$

Question 1 .... *40 (main) + 0 (bonus) points*

Consider the window operator mechanism shown in Figure 1. Let ℓ_k denote the (constant) length of link $k \in \{1, 2, 3\}$, and let ℓ_4 and ℓ_4' denote the (constant) lengths of the vectors $\overrightarrow{A_1A_2}$ and $\overrightarrow{A_2C}$, respectively. The distances a_{0x} and a_{0y} are also constants.

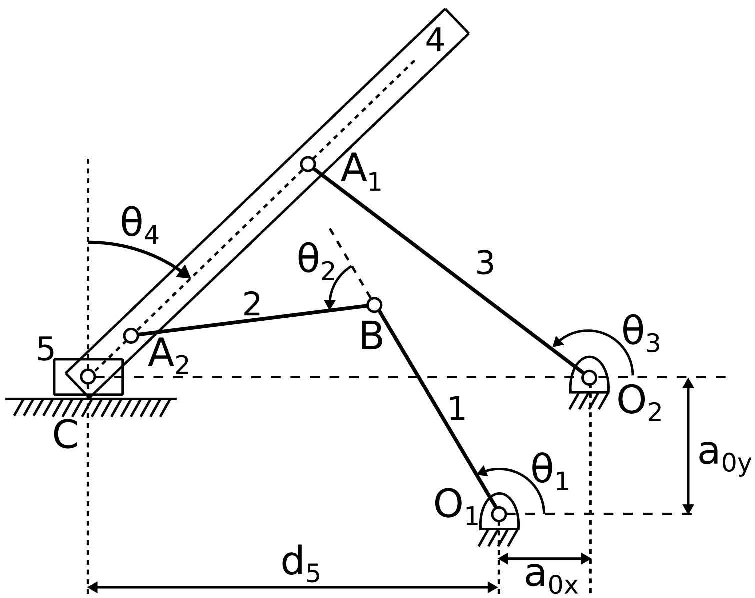


Figure 1: `Window operator linkage`

(a) [10 points] Analytically derive the loop equations in terms of the variables $\{\theta_i\}_{i=1}^{4}$, d_5, and the constant lengths.

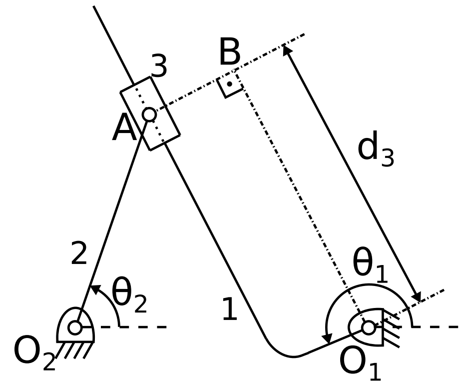
(b) [30 points] Given ${}^0\omega^3$, write down the velocity-level loop equations, *in matrix form*, used to find ${}^0\omega^1$, ${}^0\omega^2$, ${}^0\omega^4$, and ${}^0v^C$.

Question 2 . . . . *60 (main) + 0 (bonus) points*

Consider the sliding contact linkage, consisting of 3 rigid bodies, shown in the figure below. The constants pertinent to this problem are listed beside this figure.

Magnitude	Vector	Value [units]
l_0	$\left\lvert\overrightarrow{O_1O_2}\right\rvert$	1.5
l_1	$\left\lvert\overrightarrow{AB}\right\rvert$	0.5
l_2	$\left\lvert\overrightarrow{O_2A}\right\rvert$	1.25

Hint. The vector from O_1 to A can be decomposed into the sum of the vectors from O_1 to B and from B to A.



(a) [15 points] Derive the position-level loop equations for this mechanism in terms of the variables θ_1, θ_2, d_3, and the various constants l_0, l_1, and l_2. Now, if it is observed that $\theta_1 = -160°$ and $\theta_2 = 63.311830°$ then what is the value of d_3?

(b) [30 points] At this instant, if ${}^0\omega^1 = 1z_1$ rad/s, compute ${}^0\omega^2 = \dot{\theta}_2 z_2$ and ${}^1v^A = \dot{d}_3 x_1$. Here, z_1 and z_2 are the unit vectors, fixed on links 1 and 2, resp., that point out of the page; x_1 points from O_1 to B and point A is considered fixed on link 3 (or 2).

(c) [15 points] Derive the acceleration-level loop equations and use them to solve for ${}^0\alpha^2$ and ${}^1a^A$ assuming ${}^0\alpha^1 = -5z_1$ rad/s².

Question 3 .... *25 (bonus) points*

Figure 2 shows a schematic of the Oldham coupling, where r_0, r_1, and r_2 denote the lengths of the base, first, and the second links, respectively. The angle $\phi = \pi/4$ and the length $r_0 = 3/2$ in are fixed. If $\theta_1 = 5\pi/12$, $\omega_1 = \dot{\theta}_1 = -3/5$ rad/s, and $\alpha_1 = \ddot{\theta}_1 = 2$ rad/s², then perform the following analysis.

(a) [5 points (bonus)] Write down the loop equations and solve for r_1 and r_2 at this instant.

(b) [10 points (bonus)] Find $\dot{r}_1$ and $\dot{r}_2$.

(c) [10 points (bonus)] Find $\ddot{r}_1$ and $\ddot{r}_2$.

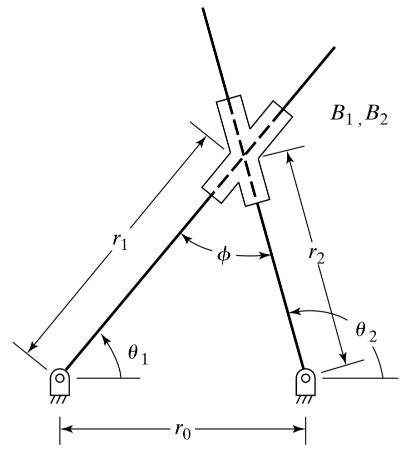


Figure 2: Oldham coupling