ME 380
Kinematics and Machine Dynamics
Midterm II

1. [35 points] Consider the mechanism shown in Figure 1. Excluding the fixed link, it consists of 5 links, one of which is a slider. Let l_k denote the length of link $k \in \{1, 2, 3\}$, and let l_4 and l'_4 denote the lengths of the vectors $\overrightarrow{A_1A_2}$ and $\overrightarrow{A_2C}$, respectively.

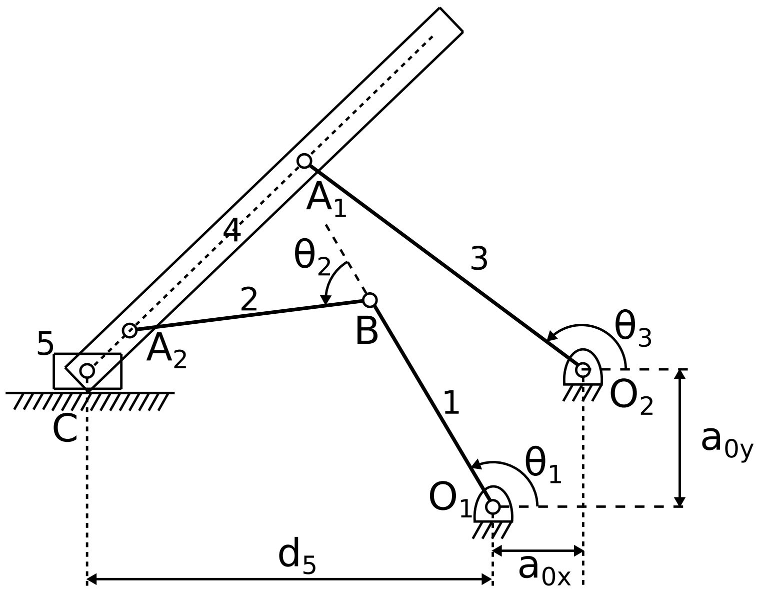


Figure 1: `Window operator linkage`

   (a) [5 points] If link 3 is the input and link 5 is the output, state which link is the coupler. Also, clearly mark the transmission angle by the quantity φ.

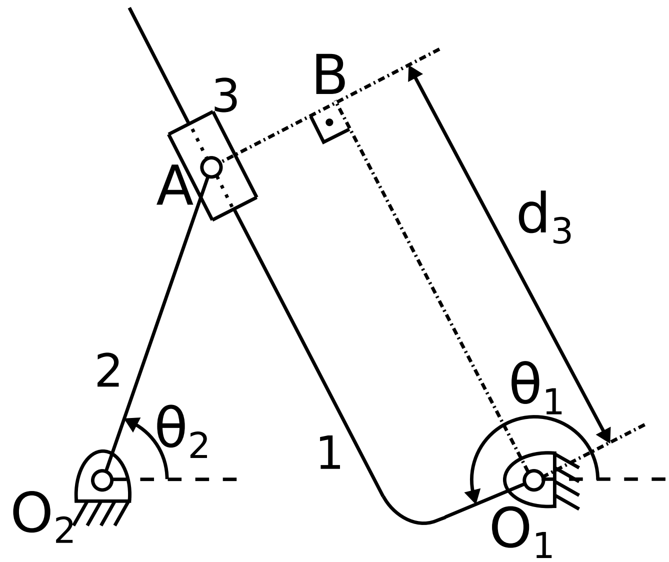
   (b) [15 points] Analytically derive the loop equations in terms of the variables $\theta_1, \theta_2, \theta_3$, the transmission angle φ, d_5, and the various constants $l_1, \ldots, l_4$, l'_4, a_{0x}, and a_{0y}.

   (c) [15 points] Given ${}^0\omega^3$, write down the velocity-level loop equations, *in matrix form*, used to find ${}^0\omega^1$, ${}^0\omega^2$, $\dot{\varphi}$, and $\dot{d}_5$,.

   (d) [5 points (bonus)] Set up an optimization problem, whose solution yields the minimum length, $l_{4_{min}}$, of $\overrightarrow{A_1A_2}$, such that the transmission angle satisfies $\frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}$. (Hint: Does not require computation, just write down an objective function and some necessary constraints.)

2. [40 points] Consider the sliding contact linkage, consisting of 3 rigid bodies, shown in the figure below. The constants pertinent to this problem are listed beside this figure.

Quantity	Endpoints	Length [units]
l_0	$\lvert O_1O_2 \rvert$	1.5
l_1	$\lvert AB \rvert$	0.5
l_2	$\lvert O_2A \rvert$	1.25



(a) [15 points] Analytically derive the position-level loop equations for this mechanism in terms of the variables θ_1, θ_2, d_3, and the various constants l_0, l_1, and l_2.

(b) [15 points] Consider an instant when $\theta_1 = -160°$. The position-level kinematics is solved with the solution $\theta_2 = 63.312°$ and $d_3 = 1.370$. At this instant, if ${}^0\omega^1 = 10z_0\,\frac{\text{rad}}{\text{s}}$ (z_0 is the axis in the world frame that points perpendicular to the plane of motion), compute ${}^0\omega^2$ and $\dot{d}_3$.

(c) [10 points] Write down the acceleration-level loop equations in *vector form* (You do not need to express them in coordinates).

3. [25 points] Consider a circular hoop of radius R, rotating about the world z-axis as shown in Figure 2. A bead traverses the circular hoop along a groove. Here are some facts to help you with the question:

- φ is the angle between x-axes of the world frame and the hoop frame.
- When $\varphi = 0$, the orientation of the hoop is identical to the world.
- θ is the angle between the z-axes of the hoop frame and the bead frame, which undergoes a simple (basic) rotation about the y-axis of the hoop frame.
- When $\theta = 0$, the orientation of the bead is identical to the hoop.
- When $\varphi = \theta = 0$, the translation of the bead, in the world frame is given by $p_{bead} = \begin{bmatrix} R & 0 & 0 \end{bmatrix}^\top$.

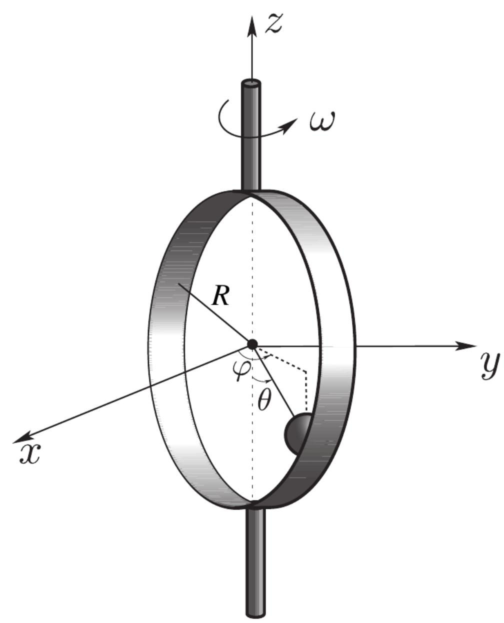


Figure 2: `Bead on a rotating hoop`

(a) [20 points] Find the (x, y, z) position of the bead in the world frame as a function of φ and θ. Also, find the velocity $(\dot{x}, \dot{y}, \dot{z})$ of the bead in the world frame as a function of φ, θ, $\dot{\varphi}$, and $\dot{\theta}$.

(b) [5 points] Suppose that $\varphi(0) = \frac{2\pi}{5}$ and the circular hoop has a constant angular speed with respect to the world, i.e., $\omega = \dot{\varphi} = \frac{\pi}{2}\ \frac{\text{rad}}{\text{s}}$. Suppose further that the bead has a constant angular speed with respect to the hoop, i.e., ${}^{hoop}\omega^{bead} = \dot{\theta} y_{hoop}$ such that $\theta(0) = \frac{\pi}{6}$ and $\theta(3) = -\frac{2\pi}{3}$. Use this information to express φ and θ as a function of time.

(c) [5 points (bonus)] Making use of parts (a) and (b) compute the position and velocity of the bead in the world frame at $t = 2.5$ sec.