

ME 380: Kinematics and Machine Dynamics

Fall 2021 / Midterm Examination #1

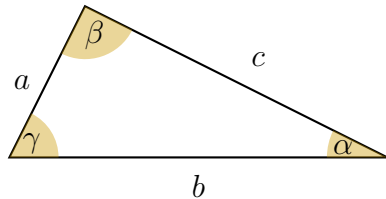
Name: _____

Angle sum and difference identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

Law of Cosines



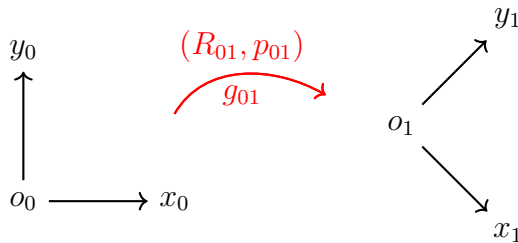
$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Rigid Body Transformations

$\mathbb{R}^3 \ni p_{01}$: position vector from o_0 to o_1 .



$$\mathbb{R}^{3 \times 3} \ni R_{01} = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

$$g_{01} = \left[\begin{array}{c|c} R_{01} & p_{01} \\ \hline 0 & 1 \end{array} \right]$$

Composition rules: $R_{ac} = R_{ab}R_{bc}$ and $g_{ac} = g_{ab}g_{bc}$.

Vector Operations

Let v and w be vectors in $\mathbb{R}^3$, then the inner (dot) product of v and w is defined by

$$v \cdot w = \|v\| \|w\| \cos \theta,$$

where θ is the angle between v and w .

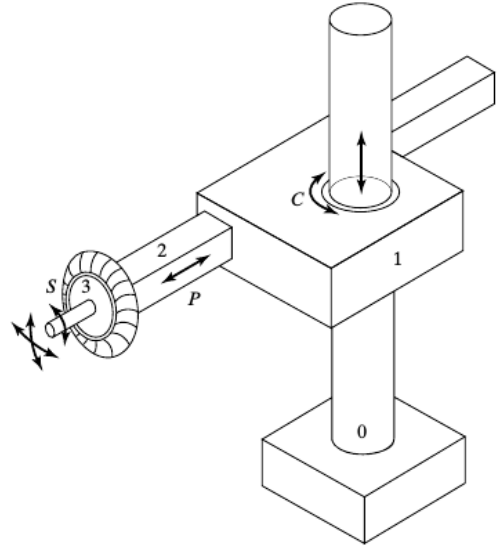
Let Σ_a be a coordinate frame, and v be a vector in $\mathbb{R}^3$. Then, the expression of v in the coordinate frame Σ_a is given by

$$v^a = [v \cdot x_a \quad v \cdot y_a \quad v \cdot z_a]^\top.$$

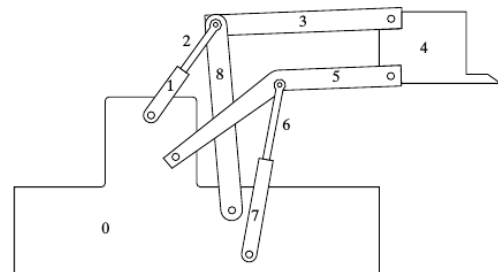
Let R_{ab} be the rotation matrix of frame Σ_b with respect to Σ_a . Then, the expressions v^a and v^b of a vector v in Σ_a and Σ_b are related by

$$v^a = R_{ab}v^b.$$

1. [25 points] This question is concerned with finding the degrees of freedom of linkages.
- (a) [10 points] Find the number of degrees of freedom for the spatial linkage below. This open-loop kinematic chain includes cylindrical, prismatic, and spherical joints.



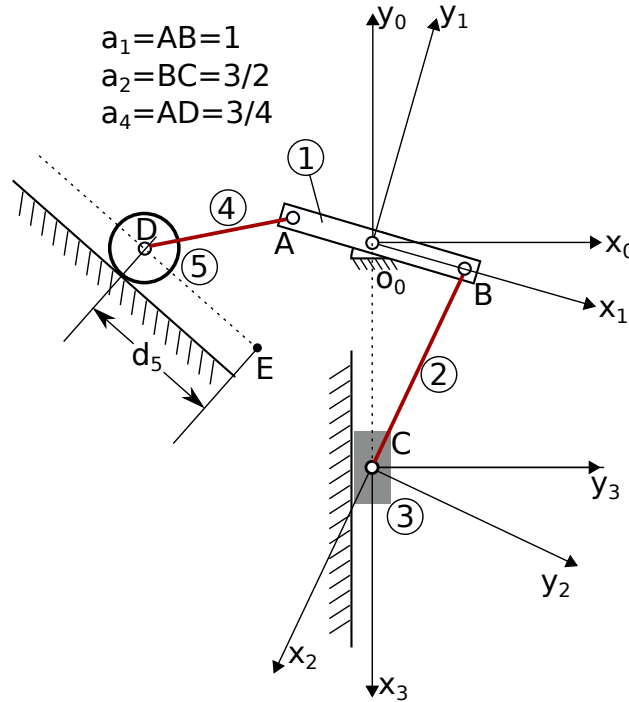
- (b) [10 points] The linkage depicted below is a schematic representation of a piece of construction machinery. It has two hydraulic cylinders (links 1 and 2, and links 6 and 7), which may be considered cylindrical pairs. The other pairs are revolute joints. Find the number of degrees of freedom if this linkage is treated as a planar linkage.



- (c) [5 points] Indicate the conditions that must be met for the planar linkage assumption to be valid in part (b).

2. [35 points] Consider the linkage shown in the figure below. The origin of the world frame, o_0 , is located in the middle of points A and B . We define the following quantities:

- θ_1 : angle between the x_0 - and x_1 -axes.
- θ_2 : angle between the x_1 - and x_2 -axes.,
- d_3 : distance along the x_3 -axis from the x_0 -axis to the y_3 -axis.
- θ_4 : angle between the unit vector from D to E and the unit vector from D to A .



- (a) [20 points] Find the rigid-body transformations between the world frame Σ_0 and the body-fixed coordinate frames Σ_1 through Σ_3 (established for you). Also find the rigid-body transformation between Σ_1 and Σ_3 .
- (b) [15 points] If $\theta_1 = -\frac{\pi}{9} = -20^\circ$, compute the position vector from o_0 to C and express it in the world frame, Σ_0 .

3. [40 points] Consider the linkage shown in Figure 1. Let $d_1 = 59.5$ in. and $d_4 = 57$ in. be the vertical and horizontal distances from the origin of the world frame, Σ_0 , to A and C , respectively. Furthermore, define the following angles:

- θ_2 : angle between x_0 -axis and the unit vector from A to B .
- θ_3 : angle between x_0 -axis and the unit vector from C to B .

Other relevant quantities are presented within the figure.

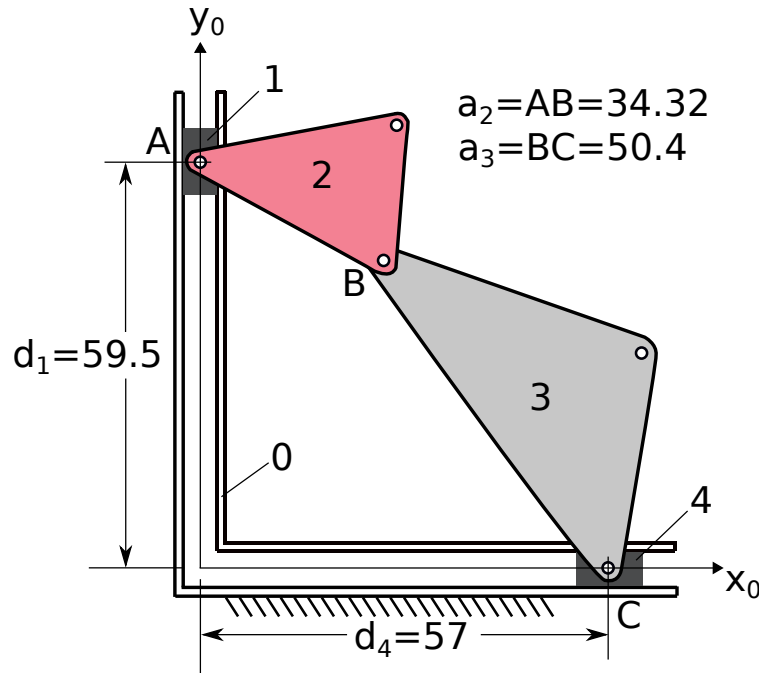


Figure 1: Dual crosshead mechanism

- [5 points] Compute the mobility of this linkage.
- [15 points] Establish coordinate frames fixed to each rigid body and compute the rigid-body transformation between the world frame (established on the figure) and each of these coordinate frames.
- [20 points] Derive the loop equations in terms of d_1 , θ_2 , θ_3 , and d_4 (the constants a_2 and a_3 are the only other quantities that may appear in these equations).
- [8 points (bonus)] Solve the loop equations you to find the values of θ_2 and θ_3 .



(a) Rendered leg test stand with leg fixed into a rotary joint on hip and foot (red) on force sensor (blue). (b) Schematic of cheetah-cub's spring-loaded leg mechanism.

Figure 2: The leg mechanism of the cheetah-cub robot from EPFL.

4. [20 points (bonus)] Figure 2 depicts the mechanism that acts as the legs of a quadruped robot, called Cheetah-cub, built in Switzerland at EPFL (see Figure 3). This mechanism mimics the leg of a real-world felidea (cat). It consists of three segments (links), the femur ($\overrightarrow{HK}$), the shank ($\overrightarrow{KA}$), and the foot ($\overrightarrow{AC}$).

The fourbar linkage that the shank is comprised of ($K - K' - A' - A$) is a geometric parallelogram, keeping the femur and the foot segments parallel.

- (a) [5 points] Compute the mobility of the leg mechanism.
- (b) [15 points] Place coordinate system, Σ_{hip} , at the hip (H) and compute the translation of the tip of the foot (C) with respect to this coordinate system when $\theta_{hip} = -15^\circ$ and $\theta_{knee} = 105^\circ$, assuming that the lengths of all segments are equal to $\ell = 150\text{mm}$.

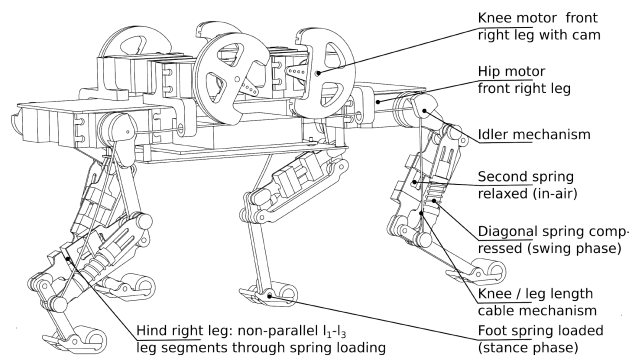


Figure 3: The Cheetah-cub robot from EPFL. The robot's view from hind right; its front is to the right. Front right and hind left leg are in swing phase, front left and hind right are in stance phase.

