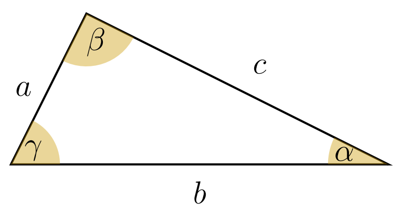
ME 380: Kinematics and Machine Dynamics
Fall 2020 / Midterm Examination #1

Name: ____

Angle sum and difference identities

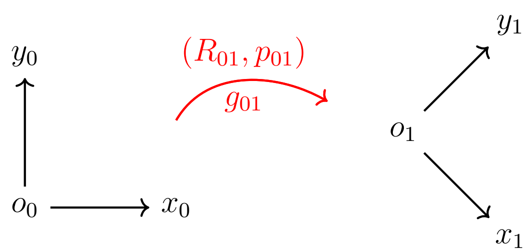
$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

Law of Cosines



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Rigid Body Transformations



$\mathbb{R}^3 \ni p_{01}$: position vector from o_0 to o_1.

$$\mathbb{R}^{3\times 3} \ni R_{01} = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

$$g_{01} = \left[\begin{array}{c|c} R_{01} & p_{01} \\ \hline 0 & 1 \end{array}\right]$$

Composition rules: $R_{ac} = R_{ab}R_{bc}$ and $g_{ac} = g_{ab}g_{bc}$.

Vector Operations

Let v and w be vectors in $\mathbb{R}^3$, then the inner (dot) product of v and w is defined by

$$v \cdot w = \|v\|\,\|w\|\cos\theta,$$

where θ is the angle between v and w.

Let Σ_a be a coordinate frame, and v be a vector in $\mathbb{R}^3$. Then, the expression of v in the coordinate frame Σ_a is given by

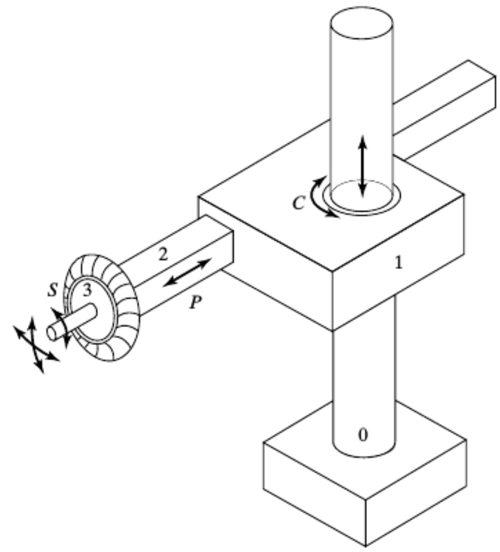
$$v^a = \begin{bmatrix} v \cdot x_a & v \cdot y_a & v \cdot z_a \end{bmatrix}^\top.$$

Let R_{ab} be the rotation matrix of frame Σ_b with respect to Σ_a. Then, the expressions v^a and v^b of a vector v in Σ_a and Σ_b are related by

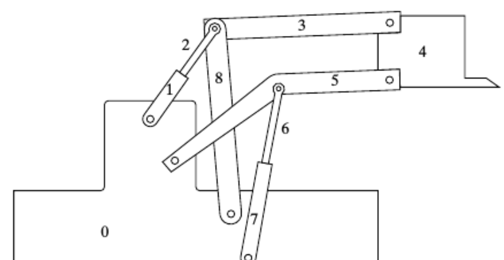
$$v^a = R_{ab}v^b.$$

1. [25 points] This question is concerned with finding the degrees of freedom of linkages.

   (a) [10 points] Find the number of degrees of freedom for the spatial linkage below. This open- loop kinematic chain includes cylindrical, prismatic, and spherical joints.



   (b) [10 points] The linkage depicted below is a schematic representation of a piece of construction machinery. It has two hydraulic cylinders (links 1 and 2, and links 6 and 7), which may be considered cylindrical pairs. The other pairs are revolute joints. Find the number of degrees of freedom if this linkage is treated as a planar linkage.

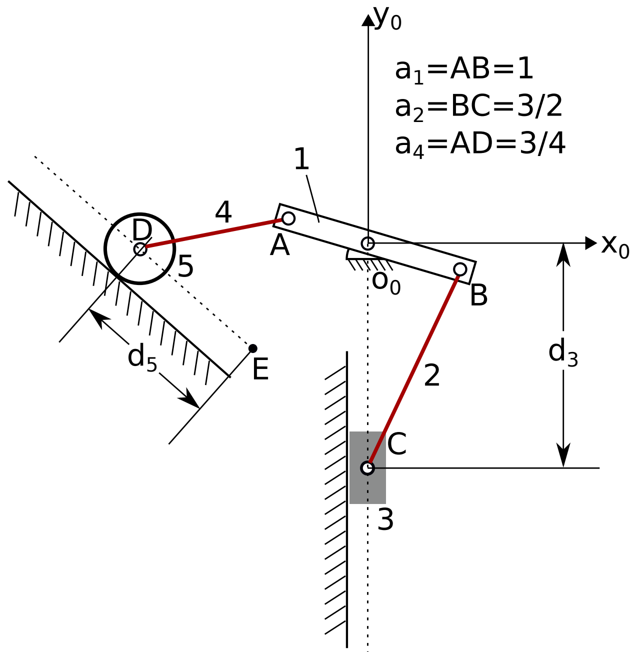


   (c) [5 points] Indicate the conditions that must be met for the planar linkage assumption to be valid in part (b).

2. [35 points] Consider the linkage shown in the figure below. Note that the origin of the world frame, o_0, is located exactly in the middle of points A and B. We define the following angles:

- θ_1: angle between the x_0-axis and the unit vector from o_0 to B.
- θ_2: angle between the unit vector from o_0 to B and the unit vector from B to C.
- θ_4: angle between the unit vector from D to E and the unit vector from D to A.

Please notice some other quantities that are presented within the figure.



(a) [20 points] Establish coordinate frames on each of the links 1 through 3. Find the rigid-body transformations (translation and rotation) between the world frame (established for you) and the body-fixed coordinate frames Σ_1 through Σ_3.

(b) [15 points] If $\theta_1 = -\frac{\pi}{9} = -20°$, compute the position vector from o_0 to C and express it in the world frame, Σ_0.

3. [40 points] Consider the linkage shown in Figure 1. Let $d_1 = 59.5$ in. and $d_4 = 57$ in. be the vertical and horizontal distances from the origin of the world frame, Σ_0, to A and B, respectively. Furthermore, define the following angles:

   - θ_2: angle between x_0 axis and the unit vector from A to B.
   - θ_3: angle between x_0 axis and the unit vector from C to B.

   Other relevant quantities are presented within the figure.

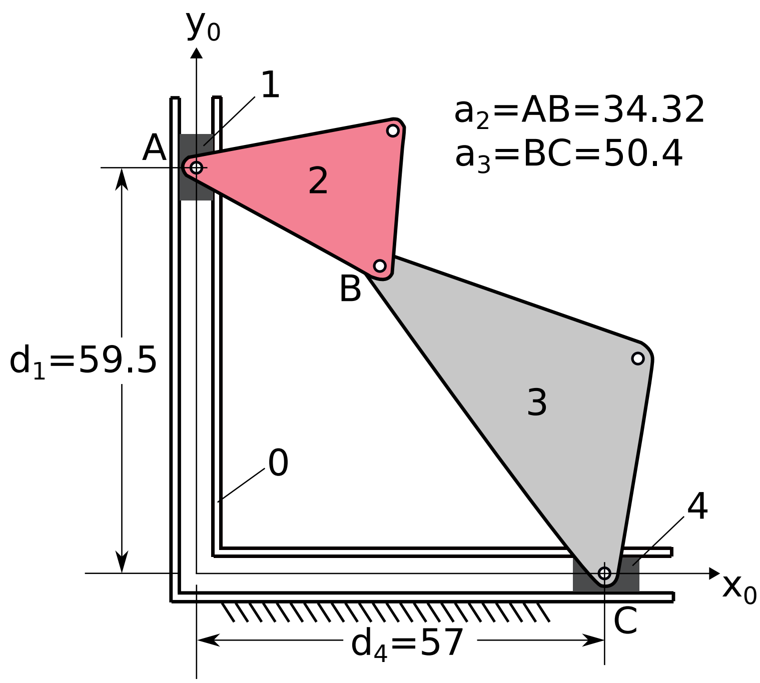


Figure 1: `Dual crosshead mechanism`

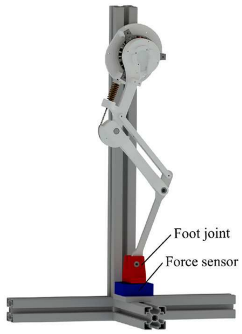
Answer the following questions about this mechanism.

(a) [5 points] Compute the mobility of this linkage.

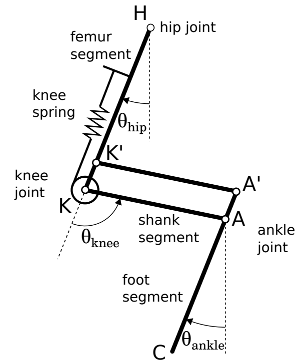
(b) [15 points] Establish coordinate frames fixed to each rigid body and compute the transformation (translation and rotation) between the world frame (established on the figure) and each of these coordinate frames.

(c) [20 points] Derive the loop equations in terms of d_1, θ_2, θ_3, and d_4.

(d) [8 points (bonus)] Solve the loop equations you derived in part (c) to find the values of θ_2 and θ_3.



(a) Rendered leg test stand with leg fixed into a rotary joint on hip and foot (red) on force sensor (blue).



(b) Schematic of cheetah-cub's spring-loaded leg mechanism.

Figure 2: `The leg mechanism of the cheetah-cub robot from EPFL.`

4. [20 points (bonus)] Figure 2 depicts the mechanism that acts as the legs of a quadruped robot, called Cheetah-cub, built in Switzerland at EPFL (see Figure 3). This mechanism mimics the leg of a real-world felidea (cat). It consists of three segments (links), the femur ($\overrightarrow{HK}$), the shank ($\overrightarrow{KA}$), and the foot ($\overrightarrow{AC}$).

   The fourbar linkage that the shank is comprised of ($K - K' - A' - A$) is a geometric parallelogram, keeping the femur and the foot segments parallel.

   (a) [5 points] Compute the mobility of the leg mechanism.

   (b) [15 points] Place coordinate system, Σ_{hip}, at the hip (H) and compute the translation of the tip of the foot (C) with respect to this coordinate system when $\theta_{\text{hip}} = -15°$ and $\theta_{\text{knee}} = 105°$, assuming that the lengths of all segments are equal to $\ell = 150\text{mm}$.

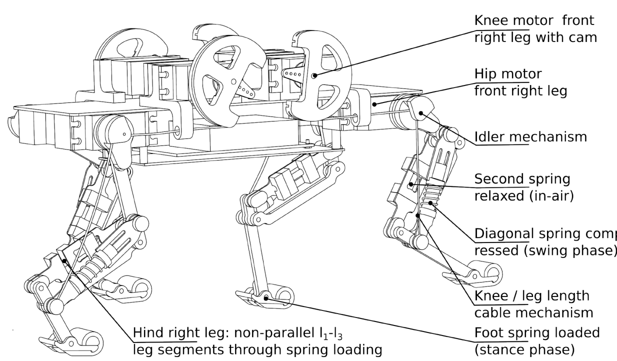


Figure 3: `The Cheetah-cub robot from EPFL.` The robot's view from hind right; its front is to the right. Front right and hind left leg are in swing phase, front left and hind right are in stance phase.