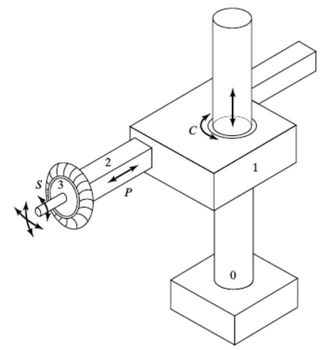
ME 380
Kinematics and Machine Dynamics
Midterm Exam I Solutions

1. [30 points] This question is concerned with finding the degrees of freedom of linkages (appears in p. 98 of Wilson and Sadler).

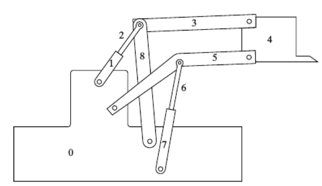
   (a) [10 points] Find the number of degrees of freedom for the spatial linkage below. This open- loop kinematic chain includes cylindrical, prismatic, and spherical joints.

   > **Solution:** There are 4 links, including the fixed link.
   > There are 1 cylindrical, 1 prismatic, and 1 spherical joints.
   >
   > $$6(4-1)-4(1)-5(1)-3(1)=6 \text{ DoF}$$



   (b) [10 points] The linkage depicted below is a schematic representation of a piece of construction machinery. It has two hydraulic cylinders (links 1 and 2, and links 6 and 7), which may be considered cylindrical pairs. The other pairs are revolute joints. Find the number of degrees of freedom if this linkage is treated as a planar linkage.

   > **Solution:** There are 9 links, including the fixed link.
   > There are 9 revolute joints, and 2 prismatic joints.
   >
   > $$3(9-1)-2(9)-2(2)=2 \text{ DoF}$$



   (c) [10 points] Indicate the conditions that must be met for the planar linkage assumption to be valid in part (b).

   > **Solution:** The planar motion applies if the motion occurs in a plane or in a set of parallel planes. The planes of motion of the links must all be parallel.

2. [35 points] Consider the linkage shown in Figure 1. Let $l_1 = |O_1A|$, $l_2 = |AB|$, $l_2' = |AC|$, $l_5 = |DE|$, $l_6 = |O_2F|$, and $l_7 = |FE|$, $l_7' = |CF|$. Answer the following questions about this mechanism (Appears in p. 74 of Norton, "An Introduction to the Synthesis and Analysis of Mechanisms and Machines").

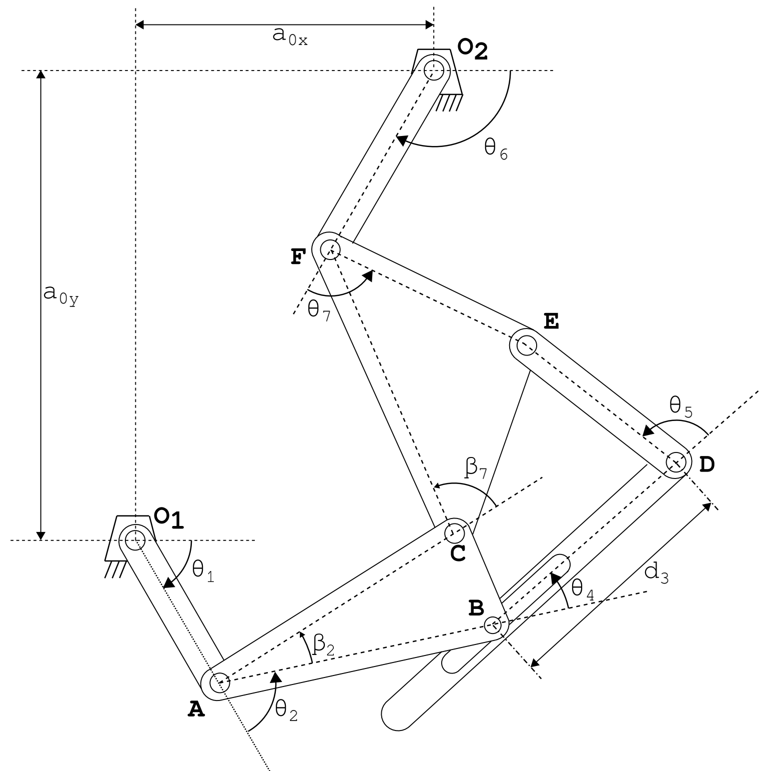


Figure 1: `Fourbar with inverted slider`

(a) [10 points] Compute the mobility of this linkage.

Solution: There are 7 moving links, 8 revolute joints, and 1 prismatic joint, thus

$$\text{DoF } = 3 \cdot 7 - 2 \cdot 8 - 2 \cdot 1 = 3.$$

(b) [20 points] Analytically derive the loop equations in terms of the variables $(\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6, \theta_7)$ and the various constants a_{0x}, a_{0y}, l_1 through l_7, and β_2, β_7.

Solution:

Loop 1: $O_1 - A - C - F - O_2 - O_1$

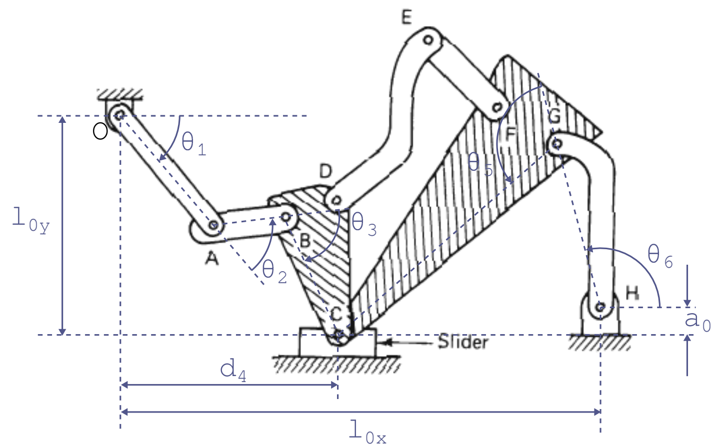
$$l_1 \begin{bmatrix} c_1 \\ s_1 \end{bmatrix} + l_2' \begin{bmatrix} \cos\theta_1 + \theta_2 + \beta_2 \\ \sin\theta_1 + \theta_2 + \beta_2 \end{bmatrix}$$
$$+ l_7' \begin{bmatrix} \cos\theta_1 + \theta_2 + \beta_2 + \beta_7 \\ \sin\theta_1 + \theta_2 + \beta_2 + \beta_7 \end{bmatrix} - l_6 \begin{bmatrix} c_6 \\ s_6 \end{bmatrix} - \begin{bmatrix} a_{0x} \\ a_{0y} \end{bmatrix} = 0,$$

Loop 2:
$O_1 - A - B - D - E - F - O_2 - O_1$

$$
\begin{aligned}
&l_1 \begin{bmatrix} c_1 \\ s_1 \end{bmatrix} + l_2 \begin{bmatrix} \cos\theta_1 + \theta_2 \\ \sin\theta_1 + \theta_2 \end{bmatrix} + d_3 \begin{bmatrix} \cos\theta_1 + \theta_2 + \theta_4 \\ \sin\theta_1 + \theta_2 + \theta_4 \end{bmatrix} \\
&+ l_5 \begin{bmatrix} \cos\theta_1 + \theta_2 + \theta_4 + \theta_5 \\ \sin\theta_1 + \theta_2 + \theta_4 + \theta_5 \end{bmatrix} \\
&- l_7 \begin{bmatrix} \cos\theta_6 + \theta_7 \\ \sin\theta_6 + \theta_7 \end{bmatrix} - l_6 \begin{bmatrix} c_6 \\ s_6 \end{bmatrix} - \begin{bmatrix} a_{0x} \\ a_{0y} \end{bmatrix} = 0.
\end{aligned}
$$

3. [40 points] Consider the linkage shown in the figure below. Answer the following questions related to this mechanism (Appears in p. 45 of Sandor, "Advanced Mechanism Design").

Quantity	Endpoints	Length [mm]
l_1	$\lvert OA \rvert$	100
l_2	$\lvert AB \rvert$	50
l_3	$\lvert BC \rvert$	90
l_5	$\lvert GC \rvert$	200
l_6	$\lvert HG \rvert$	125
l_{0x}	shown	350
l_{0y}	shown	160
a_0	shown	15



(a) [7 points] Compute the mobility of this linkage and identify the various sub-components of this mechanism as various simpler mechanisms we have discussed in class.

> **Solution:** There are 8 moving rigid bodies, 10 revolute joints, and 1 prismatic joint. Thus,
> $$\text{DoF} = 3 \cdot 8 - 2 \cdot 10 - 2 \cdot 1 = 2.$$
> The three moving bodies that consist of the rightmost curly link, the rightmost shaded link, and the slider form a slider-crank. The four bodies that consist of the two shaded links and the links that connect these links (via joints at D, E, and F) constitute a four-bar linkage.

(b) [13 points] Derive the position-level loop equations for this mechanism.

> **Solution:** Loop 1:
> O-A-B-C-O
> $$l_1 \begin{bmatrix} c_1 \\ s_1 \end{bmatrix} + l_2 \begin{bmatrix} c_{12} \\ s_{12} \end{bmatrix} + l_3 \begin{bmatrix} c_{123} \\ s_{123} \end{bmatrix} + \begin{bmatrix} -d_4 \\ l_{0y} \end{bmatrix} = 0,$$
> Loop 2:
> H-G-C-H
> $$\begin{bmatrix} l_{0x} \\ a_0 \end{bmatrix} + l_6 \begin{bmatrix} c_6 \\ s_6 \end{bmatrix} + l_5 \begin{bmatrix} c_{56} \\ s_{56} \end{bmatrix} - \begin{bmatrix} d_4 \\ 0 \end{bmatrix} = 0.$$

(c) [20 points] Use vector cross product method to solve for the position-level kinematics at an instant when $\theta_1 = -\frac{5\pi}{16}$ rad $= 56.25°$ and $d_4 = 175$ mm. (solve for θ_2, θ_3, θ_5, and θ_6).

Solution: First we solve for θ_2 and θ_3 via case 3 of the vector cross product method, where we set $w = \overrightarrow{CA}$, $u = \overrightarrow{AD}$ and $v = \overrightarrow{BC}$. We know that $|u| = l_2$ and $|v| = l_3$.

We then repeat the same process, but define $w = \overrightarrow{CH}$, $u = \overrightarrow{HG}$, and $v = \overrightarrow{GC}$. We know that $|u| = l_6$ and $|v| = l_5$. The overall solution is

Unknowns	Radians	Degrees
θ_2	0.75643895	43.34076°
θ_3	−0.44177351	−25.31176°
θ_5	2.08920670	119.70273°
θ_6	1.80430062	103.37881°