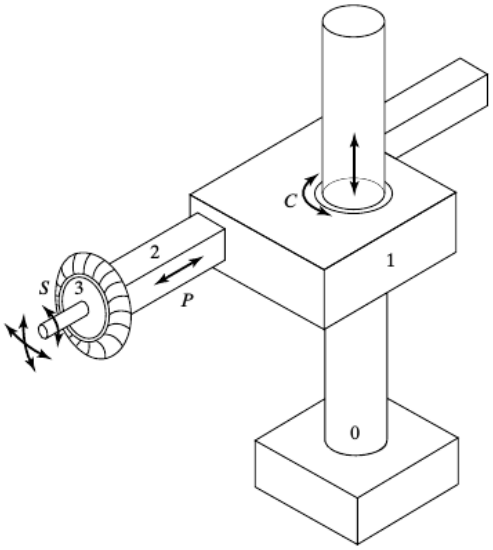
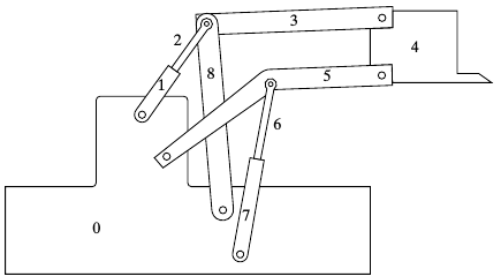
ME 380
Kinematics and Machine Dynamics
Midterm

1. [25 points] This question is concerned with finding the degrees of freedom of linkages.

   (a) [10 points] Find the number of degrees of freedom for the spatial linkage below. This open- loop kinematic chain includes cylindrical, prismatic, and spherical joints.



   (b) [10 points] The linkage depicted below is a schematic representation of a piece of construction machinery. It has two hydraulic cylinders (links 1 and 2, and links 6 and 7), which may be considered cylindrical pairs. The other pairs are revolute joints. Find the number of degrees of freedom if this linkage is treated as a planar linkage.



   (c) [5 points] Indicate the conditions that must be met for the planar linkage assumption to be valid in part (b).

2. [35 points] Consider the quick-return shaper mechanism shown in Figure 1. Excluding the fixed link, it consists of 5 links, one of which is the slider, as labelled in the figure. Let l_k denote the length of link $k \in \{1, 3, 4\}$.

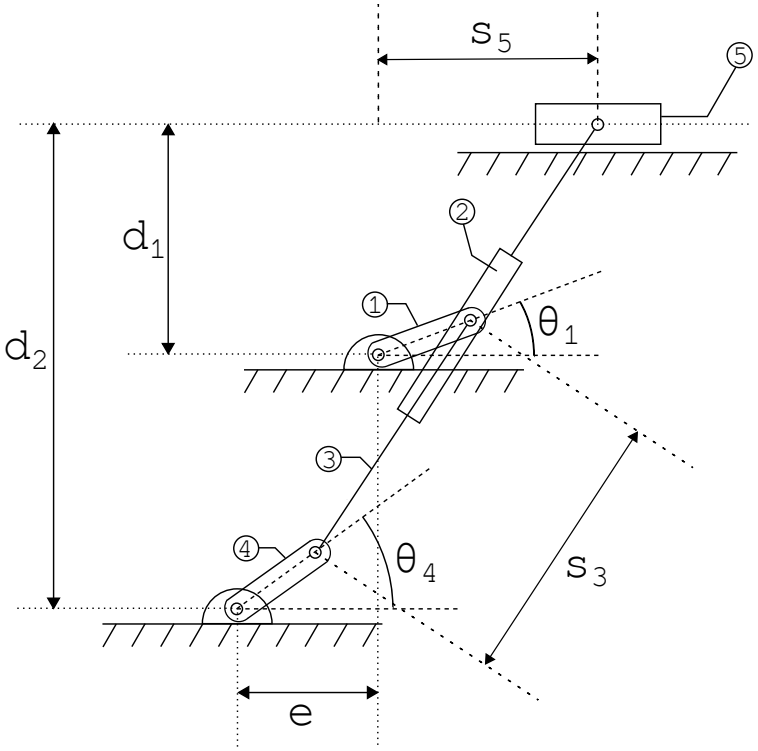


Figure 1: `Quick-return shaper mechanism`

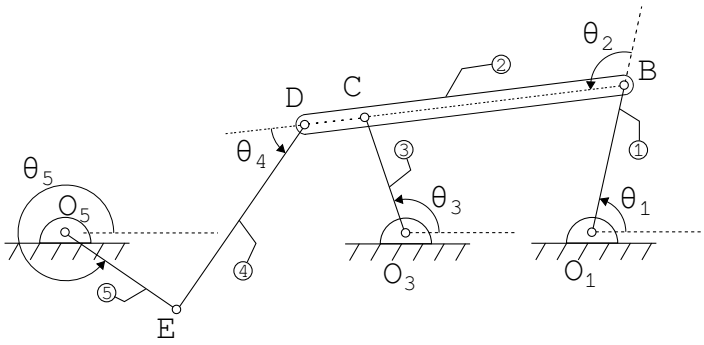
(a) [7 points] If link 1 is the input and link 5 is the output, state which link is the coupler. Also, clearly mark the transmission angle by the quantity φ.

(b) [18 points] Analytically derive the loop equations in terms of the variables θ_1, θ_4, s_3, s_5, the transmission angle φ, and the various constants d_1, d_2, e, l_1, l_3, and l_4.

(c) [10 points] Set up an optimization problem, whose solution yields the minimum length $l_{3_{min}}$ of link 3, such that the transmission angle satisfies $\frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}$.

3. [40 points] Consider the double four-bar mechanism shown in the figure below. Excluding the fixed link, it consists of 5 links, all connected together with revolute joints.

Quantity	Endpoints	Length [in]
l_0	$\lvert O_3O_1\rvert$	1
l_0'	$\lvert O_5O_3\rvert$	4
l_1	$\lvert O_1B\rvert$	3
l_2	$\lvert BC\rvert$	2.25
l_2'	$\lvert BD\rvert - l_2$	1
l_3	$\lvert O_3C\rvert$	2
l_4	$\lvert DE\rvert$	2.75
l_5	$\lvert O_5E\rvert$	1



(a) [8 points] Derive the position-level loop equation for this mechanism.

(b) [16 points] Use vector cross product method to solve for the position-level kinematics at an instant when $\theta_1 = \frac{5\pi}{12}$ rad (solve for θ_2, θ_3, θ_4, and θ_5).

(c) [16 points] At this instant, if ${}^0\omega^1 = 10\hat{e}_3 \frac{rad}{s}$, what is ${}^0\omega^5$?

4. [15 points (bonus)] Consider the planar two-link manipulator depicted in Figure 2. Suppose that the length l_1 of link 1 is unity.

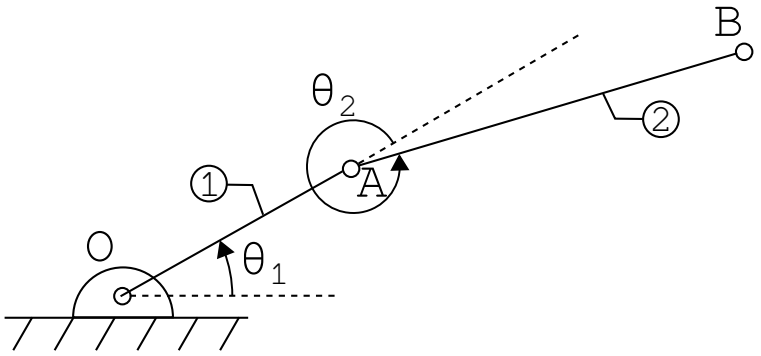


Figure 2: `Planar two-link manipulator`

A point Q moves from A towards B along link 2 with a constant rate 1 $\frac{m}{s}$ while link 1 rotates from $\theta_1(0) = \frac{\pi}{6}$ to $\theta_1(3) = \frac{2\pi}{3}$ at a constant rate in 3 seconds. During this time, link 2 rotates from $\theta_2(0) = -\frac{\pi}{12}$ to $\theta_2(3)$ to $\theta_2(3) = -\frac{\pi}{3}$ at a constant rate. Find the position of point Q as a function of time, expressed in the world frame. What is its position at $t = 2.5$ seconds?