# ME 380: Kinematics and Machine Dynamics
# Linear Algebra and Coordinate Frames

## 1 How to use this assignment

In this assignment, students will be reminded basic facts about linear algebra. They will use these facts to define and manipulate coordinate frames in 3D-space.

- The instructor will provide questions and provide intermittent help.
- This activity provides the opportunity for the students to learn / recall important definitions and facts on linear algebra. They will use these tools to define and perform operations on coordinate frames.
- Students will spend approximately 30 minutes on this activity.
- This activity will be submitted and graded.

## 2 Learning objectives

After completing this assignment, students will be able to

- Students will be able to perform dot and cross multiplication, take inner products, compute norms, multiply and invert matrices.
- They will define coordinate frames and learn to express vectors in various such frames.

## 3 Assignment completion instructions

1. Planning
   - What will the instructor need before the activity?
   - What will the student need before the activity?
     - Pencil and paper
     - Calculator
2. Source material(s) for the activity
   - Lecture notes
   - Resources placed on Piazza.
3. Introduction and warm-up practice activity for the students
   - Introduction and instructions
     - A vector contains a magnitude and a direction.

  - A coordinate frame in 3D-space is comprised of three *orthonormal* vectors in $\mathbb{R}^3$.
  - A vector has different *components* in different coordinate frames.
- Warm-up and practice
  - Brainstorm
  - Think, pair, share:
    * We use the dot (inner) product to find components of a vector on another vector.
    * Matrix multiplication is not commutative: $AB \neq BA$ unless $A$ and $B$ are special.
    * A *rotation* matrix is an *orthogonal* matrix with unit *determinant*.

4. Main activity
   - Students can submit a soft-copy via email or hand-in a hard-copy per group.
   - This activity graded.
5. Wrap-up / reflection
   - Peer review
     - Have a neighboring group review your work.
     - Neighboring group should provide opinion about whether or not the solution and its development are correct.
     - After the peer review is done, the owner group tries to incorporate the suggestions and improve the solution.
   - 3-2-1 Countdown
   - One-minute paper

# 4 Conclusion and moving forward

This activity is aimed to teach the students the necessary linear algebra background to define coordinate frames and manipulate vectors in multiple such frames.

# 5 Grading criteria

- Preparation
- Critical thinking
- Technical expertise
- Adherence to assignment guidelines
- Timeliness

### 5.1 Peer reviewer notes

1. Check if various vector-vector, matrix-vector, or matrix-matrix multiplications are well-defined and correctly performed.
2. Check if the angle between two vectors are correctly computed.
3. Check if vectors are expressed correctly in different coordinate frames.

### 5.2 Preparation and content

Students can prepare for this assignment by reviewing the lecture notes. Specifically, students should review . . .

### 5.3 Creativity and communication

Students should be able to explain their thinking process to a fellow student in order to make sure that they have learnt the material.

### 5.4 Critical thinking

Students should think about the fact that while points and vectors are inherent objects in a Euclidean space, their representations in different coordinate frames are additional constructs and depend on the particular coordinate frame.

### 5.5 Engagement with topic

Every student should be able to and attempt to explain how they think about the problem to a fellow student.

### 5.6 Technical expertise

Linear algebra

# 6 Questions

1. [15 points] For each of the following set of link lengths

    (i) $l_0 = 1$, $l_1 = 3.25$, $l_2 = 1.5$, $l_3 = 3.5$

    (ii) $l_0 = 2.5$, $l_1 = 1$, $l_2 = 2.5$, $l_3 = 2$

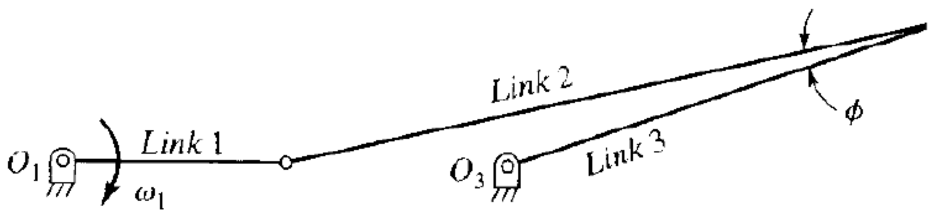


**Figure 1** – Four–bar Mechanism

    (a) [5 points] Determine whether the links can actually form a mechanism with the dimensions given.

    (b) [5 points] If a mechanism exists, identify the motion relationships (crank-rocker, drag-link, double-rocker, triple-rocker or change point)

    (c) [5 points] Show the limiting positions if the mechanism is a crank-rocker, double-rocker, or triple-rocker mechanism.

2. [20 points] Suppose that there are two coordinate frames $\Sigma_0 = \{x_0, y_0, z_0\}$ and $\Sigma_1 = \{x_1, y_1, z_1\}$. They are related by the equations

$$\begin{aligned} x_0 &= x_1 + y_1 + 3z_1 \\ y_0 &= 2x_1 - y_1 + 4z_1 \\ z_0 &= 3x_1 + 5z_1 \end{aligned}$$

Notice that these coordinate frames are not related by a rigid body transformation (why?)

    (a) [10 points] Find the transformation matrix $T$ that takes $\Sigma_0$ onto $\Sigma_1$.

    (b) [10 points] Suppose the angular velocity $\omega$ of a rigid body is expressed in $\Sigma_1$ by $\omega = \begin{bmatrix}-1 & 2 & 3\end{bmatrix}^\top$. What is its representation in $\Sigma_0$?

3. [20 points] Do the following computations

    (a) [5 points] Find unit vector along the vector $v = \begin{bmatrix}18 & -11 & 7\end{bmatrix}^\top$.

    (b) [5 points] Compute the angle between the vectors $v = \begin{bmatrix}16 & 10 & -10\end{bmatrix}^\top$ and $w = \begin{bmatrix}-17 & 9 & -12\end{bmatrix}^\top$.

    (c) [5 points] Find a unit vector, orthogonal to both $v = \begin{bmatrix}-1 & 1 & 0\end{bmatrix}$ and $w = \begin{bmatrix}-3 & 4-5\end{bmatrix}^\top$.

(d) [5 points] Is the following set of vectors *linearly independent*? Explain your answer.

$$e_1 = \begin{bmatrix} -5 & \frac{5}{9} & -\frac{35}{9} \end{bmatrix}^\top,$$
$$e_2 = \begin{bmatrix} \frac{25}{9} & \frac{5}{3} & -\frac{5}{9} \end{bmatrix}^\top,$$
$$e_3 = \begin{bmatrix} -\frac{185}{9} & 5 & -\frac{115}{9} \end{bmatrix}^\top.$$

4. [20 points (bonus)] Consider the vector space over $\mathbb{R}$ of polynomials in one indeterminate $x$ with real coefficients of degree 2. We denote this space by $\mathbb{R}[x]_2$. Addition and scalar multiplication are defined by

$$(f+g)(x) = f(x) + g(x), \quad (\alpha f)(x) = \alpha f(x).$$

for all $f, g \in \mathbb{R}[x]_2$, and $\alpha \in \mathbb{R}$. We think of these polynomials as functions from $[0,1]$ to $\mathbb{R}$ and let the inner product between two elements of this space be defined by

$$\langle f, g \rangle = \int_0^1 f(x) g(x) \, \mathrm{d}x, \tag{1}$$

whenever $f, g \in \mathbb{R}[x]_2$. Let $\{f_1, f_2, f_3\}$ be an orthonormal basis of $\mathbb{R}[x]_2$. Suppose that $f_1(x) = 1$ and $f_2(x) = \sqrt{3} - 2\sqrt{3}x$ are given.

(a) [10 points] Find $f_3$.

(b) [10 points] Consider the correspondence between $\mathbb{R}[x]_2$ and $\mathbb{R}^3$, given by $f_i \leftrightarrow e_i$, for $i = 1, 2, 3$, where $e_i$ is the $i^{\text{th}}$ standard basis vector. Find a matrix $Q \in \mathbb{R}^{3\times 3}$ such that the inner product on $\mathbb{R}[x]_2$, given by equation (1), matches the $Q$-inner product on $\mathbb{R}^3$, given by

$$\langle v, w \rangle = v^\top Q w. \tag{2}$$

for all $v, w \in \mathbb{R}^3$.