ME 380
Kinematics and Machine Dynamics
Homework VI

1. [25 points] For the planar mechanism shown in Figure 1, draw the free body diagram for each link. Write down equations of motion for each link. Identify number of equations and all the unknowns. Put equations in a matrix form.

   Assume uniform links with center of mass located at the link centers. Gravity is out of plane. Links have masses m_i and inertias J_i, $i = 1, \ldots, 5$ about their centers of mass, in the direction perpendicular to the plane. Assume that kinematic analysis has already been performed and all accelerations are known. No friction exists at the joints. (Hint: To simplify free body diagrams at the multiple link junction, consider point J as an auxiliary rigid body with no mass.)

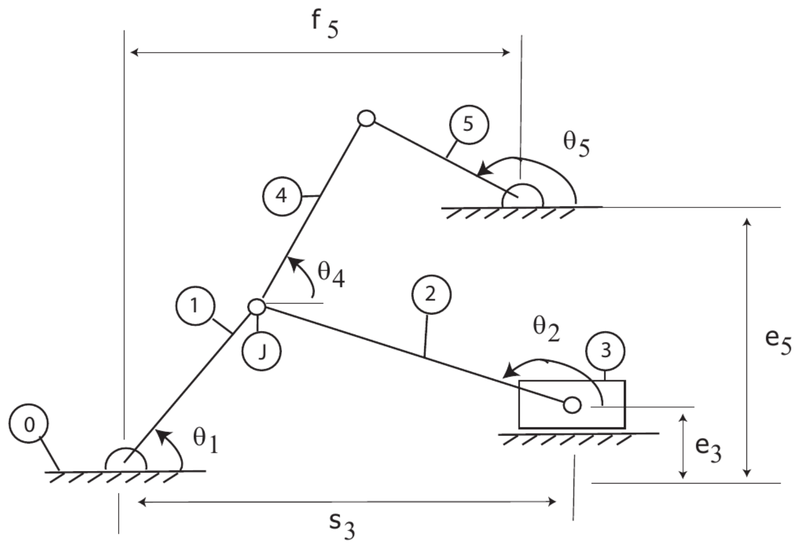

Figure 1: Planar Mechanism

Refer to Figure 2 for questions 2, 3, and 4. The link lengths are given by $l_0 = 0.25\,\text{m}$, $l_1 = l_4 = 0.3\,\text{m}$, $l_2 = l_3 = 0.35\,\text{m}$. Each link is made of homogeneous material, so assume that the center of mass of each link is at its geometric center. The inertial constants are given by $m_1 = m_4 = 0.15\,\text{kg}$, $m_2 = m_3 = 0.1\,\text{kg}$ and $I_1 = I_4 = 1.5 \times 10^{-3}\,\text{kg} \cdot \text{m}^2$, $I_2 = I_3 = 2 \times 10^{-3}\,\text{kg} \cdot \text{m}^2$. Gravity acts downwards within the plane.

2. [25 points] Perform position-level kinematic analysis for the pantograph depicted in Figure 2. Follow this analysis up by performing a static-force analysis. Find the values of the motor torque $\tau = \begin{bmatrix} \tau_1 & \tau_4 \end{bmatrix}^\top$ necessary to "cancel the gravity" (i.e. keep the mech-

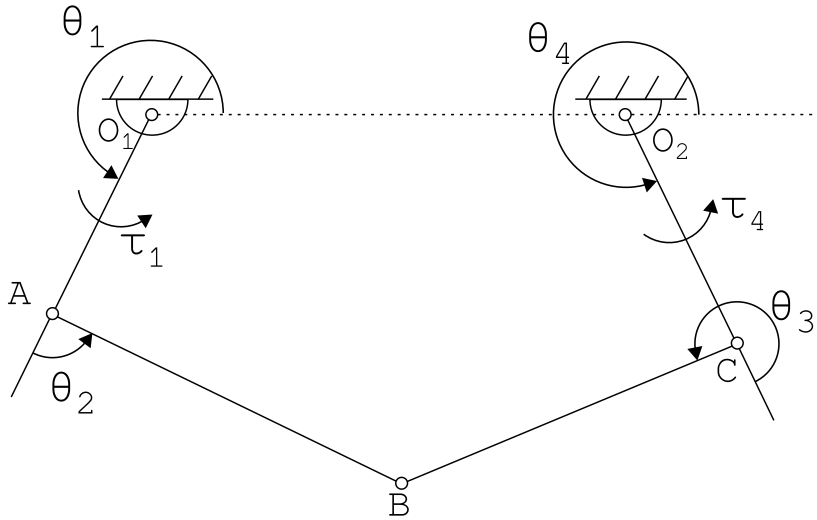

Figure 2: `Pantograph`

anism stationary) when $\begin{bmatrix}\theta_1 & \theta_4\end{bmatrix}^\top = \begin{bmatrix}240^\circ & 310^\circ\end{bmatrix}^\top$. (Notice that you first need to find what θ_2 and θ_3 correspond to these values of θ_1 and θ_4.)

3. [25 points] Complete the kinematic analysis of the pantograph by perfoming a velocity- and acceleration-level analysis. Given that $\begin{bmatrix}\theta_1 & \theta_4\end{bmatrix}^\top = \begin{bmatrix}240^\circ & 310^\circ\end{bmatrix}^\top$ and $\begin{bmatrix}\dot\theta_1 & \dot\theta_4\end{bmatrix}^\top = \begin{bmatrix}-15 & 5\end{bmatrix}^\top \frac{\text{rad}}{\text{s}}$, at an instant in time, find $\begin{bmatrix}\theta_2 & \theta_3\end{bmatrix}^\top$ and $\begin{bmatrix}\dot\theta_2 & \dot\theta_3\end{bmatrix}^\top$. Additionally, find the angular acceleration of all links at this instant if $\tau = \begin{bmatrix}\tau_1 & \tau_4\end{bmatrix}^\top = \begin{bmatrix}1 & -1\end{bmatrix}^\top$ Nm.

4. [25 points (bonus)] (Strongly recommended as this will help with your project immensely)

   Assume that the motor torques equal identically to $\tau = \begin{bmatrix}-0.45 & 0.45\end{bmatrix}^\top$. Suppose at the initial time ($t = 0$) the mechanism's state is $\begin{bmatrix}\theta_1 & \theta_4 & \dot\theta_1 & \dot\theta_4\end{bmatrix} = \begin{bmatrix}240^\circ & 310^\circ & 0 & 0\end{bmatrix}$. Write a Julia or Matlab program that simulates the system for 20 seconds (finds θ_1, θ_4, $\dot\theta_1$, and $\dot\theta_4$ for all time $t \in [0, 20]$ using Newton-Euler equations). You may use the Julia package `DifferentialEquations`, Matlab's `ode45` function or simply implement a backward Euler integration scheme for numerical integration. Present the simulation results by plotting θ_1, θ_4, $\dot\theta_1$, and $\dot\theta_4$ as a function of time t.