ME 380
Kinematics and Machine Dynamics
Homework III

1. [15 points] Let r_1, r_2, and r_3 be vectors on a plane such that

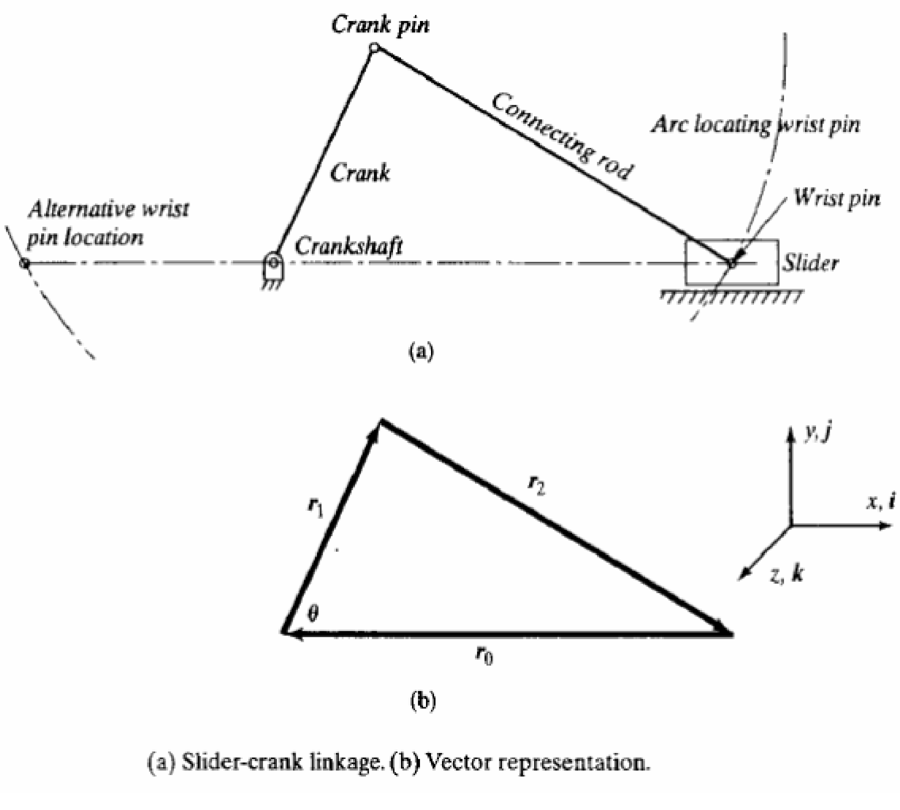
$$r_1 + r_2 + r_3 = 0$$

If we additionally know that

$$\frac{r_{1y}}{r_{1x}} = 2, \quad \frac{r_{2y}}{r_{2x}} = \frac{1}{2}, \quad r_3 = \begin{bmatrix} 50 & 75 \end{bmatrix}^\top$$

Find r_1 and r_2 by using the vector cross product method.

2. [15 points] An inline slider-crank linkage has a crank length l_1 and a connecting rod length $l_2 = \frac{3}{2}l_1$. Find the connecting rod and the slider position (in terms of l_1) when the crank angle is $\theta = \frac{2\pi}{9}$ by using analytical vector methods.



(a) Slider-crank linkage. (b) Vector representation.

3. [15 points] Plot the slider position (with Julia or Matlab) versus θ for an inline slider-crank linkage for which the ratio of the connecting rod to crank length is $\frac{3}{2}$. Let $\theta \in [0, 2\pi]$, discretized uniformly with step length $\delta\theta = \frac{\pi}{9}$.

4. [15 points] The link lengths of a planar four-bar mechanism are $l_0 = 120$, $l_1 = 60$, $l_2 = 140$, $l_3 = 80$. Find the orientation of links 2 and 3 when the internal angle between the crank and the fixed link is $\frac{\pi}{6}$ and the linkage is in the open phase (i.e. the coupler does not cross the fixed link). Use the vector cross-product method.

5. [20 points (bonus)] *Properties of rotation matrices*
   Let $R \in SO(3)$ be a rotation matrix generated by rotating about a unit vector ω by θ radians. That is, R satisfies $R = \exp(\omega_\times \theta) = I + (\sin\theta)\omega_\times + (1 - \cos\theta)\omega_\times^2$.

   (a) [5 points] Show that the eigenvalues of $\omega_\times$ are 0, i, and $-i$, where $i = \sqrt{-1}$. What is the eigenvector whose eigenvalue is 0?

   (b) [10 points] Show that the eigenvalues of R are 1, $e^{i\theta}$, and $e^{-i\theta}$. What is the eigenvector whose eigenvalue is 1?

   (c) [5 points] Let $R = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}$ be a rotation matrix. We know that $|R| = r_1 \cdot (r_2 \times r_3)$. Show that $r_1 \cdot (r_2 \times r_3) = r_2 \cdot (r_3 \times r_1) = r_3 \cdot (r_1 \times r_2)$.