ME 380
Kinematics and Machine Dynamics
Homework II

1. [15 points] For each of the following set of link lengths

   (i) $l_0 = 1$, $l_1 = 3.25$, $l_2 = 1.5$, $l_3 = 3.5$

   (ii) $l_0 = 2.5$, $l_1 = 1$, $l_2 = 2.5$, $l_3 = 2$

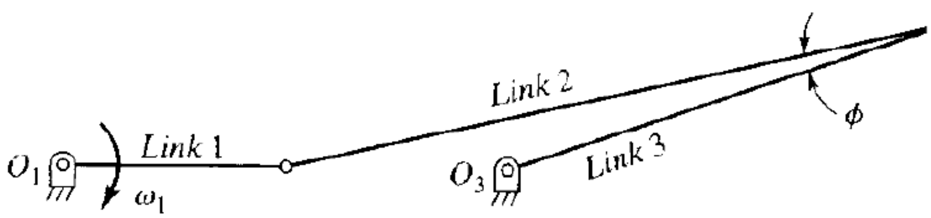

Figure 1 – Four–bar Mechanism

   (a) [5 points] Determine whether the links can actually form a mechanism with the dimensions given.

   (b) [5 points] If a mechanism exists, identify the motion relationships (crank-rocker, drag-link, double-rocker, triple-rocker or change point)

   (c) [5 points] Show the limiting positions if the mechanism is a crank-rocker, double-rocker, or triple-rocker mechanism.

2. [18 points] Find the range of values of the unknown link length, l_3, for $l_0 = 120$, $l_1 = 220$, $l_2 = 80$, if the linkage will theoretically act as

   (a) [3 points] a crank-rocker mechanism (where link 1 rotates through 2π)

   (b) [3 points] a drag-link mechanism

   (c) [3 points] a double-rocker mechanism

   (d) [3 points] a change-point mechanism

   (e) [3 points] a Grashof mechanism

   (f) [3 points] a triple-rocker mechanism

3. [15 points] Let $l_0 = 150$mm, $l_1 = 110$mm, $l_2 = 150$mm, find the length of the link l_3 so that the linkages form a crank-rocker mechanism and the transmission angle falls between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

4. [15 points] This problem refers to an offset slider-crank mechanism depicted on the next page.

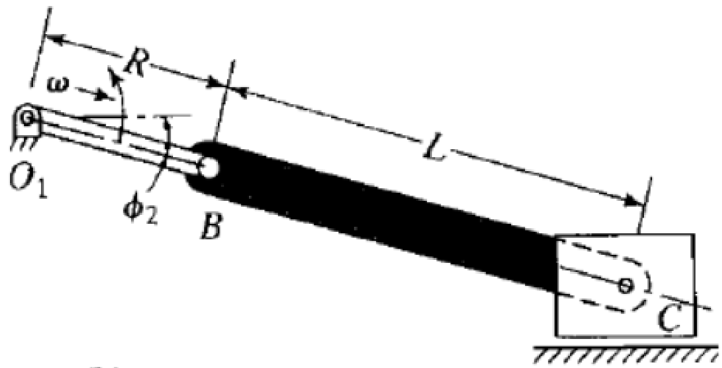

Figure 2 – Offset Slider–Crank Mechanism

The transmission angle is limited to the range between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ (i.e., the angle between the connection rod and the slider path must fall between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$). The crank length is 400mm, and the offset is 200mm; find the minimum connecting rod length.

5. [20 points] Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = \begin{bmatrix} x_1^2 + x_2^2 - \alpha^2 \\ x_1 x_2 - \beta \end{bmatrix}$$

parameterized by $\alpha > 0$ and $\beta > 0$. Define $x_0 := \begin{bmatrix} \frac{2}{8} & \frac{3}{8} \end{bmatrix}^\top$.

(a) [10 points] Suppose $(\alpha, \beta) = (\frac{\sqrt{3}}{2}, \frac{1}{4})$.
Use your favorite programming language to code the Newton-Raphson algorithm. Use this algorithm to find an x^* such that $f(x^*) \approx 0$ starting from the initial guess x_0. You can assume the algorithm has "converged" whenever $\|f(x^*)\| \leq \epsilon = 10^{-6}$.

(b) [10 points] Find the analytic solutions of the equation $f(x) = 0$ using algebra. Compare your solutions with the solution you obtained in part (a).

(c) [5 points (bonus)] Use a built-in nonlinear solver (examples: Julia's `nlsolve` command contained in the package `NLsolve` or Matlab's `fsolve` command) to find an x^* such that $f(x^*) \approx 0$ starting from the initial guess x_0. Compare the result with what you found in part (a).