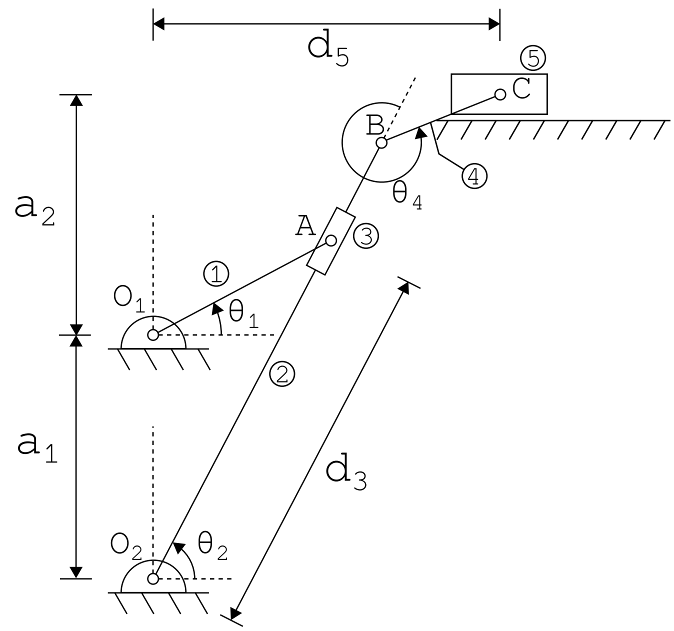
ME 380: Kinematics and Machine Dynamics
Final Examination

1. [20 points] Consider the Whitworth quick-return mechanism shown the figure below. The lengths of links 1, 2, and 4 are given by l_1, l_2, and l_4. The position of link 3 along link 2 is given by d_3 and the horizontal position of link 5 is given by d_5.

Quantity	Length [m]
a_1	1
a_2	1
l_1	$\frac{3}{4}$
l_2	$\frac{7}{4}$
l_4	$\frac{2}{3}$



   (a) [10 points] Given θ_1, write down the loop equations used to solve for θ_2, d_3, θ_4, and d_5. Express the vector form of the equations in the world ($\Sigma_0 = \{x_0, y_0, z_0\}$) frame.

   (b) [10 points] Given ${}^0\omega^1 = \dot{\theta}_1 z_0$, write down the equations to find ${}^0\omega^2 = \dot{\theta}_2 z_0$, $\dot{d}_3$, ${}^0\omega^4 = \dot{\theta}_4 z_0$, and $\dot{d}_5$ in matrix form.

   (c) [10 points (bonus)] If, at this instant, the position-level kinematic analysis is performed with the solution $(\theta_1, \theta_2, d_3, \theta_4, d_5) = (30°, 64.715°, 1.521, -25.923°, 1.267)$, and $\dot{\theta}_1 = -\frac{1}{2}\ \frac{\text{rad}}{\text{s}}$, find $\dot{d}_5$.

2. [40 points] Consider the toggle linkage shown in Figure 1. The masses of all of the links are uniformly distributed. Gravity acts in the downward direction within the plane of operation. Assume that the center of mass of link 5 is at the point B. In your solutions, make sure to use the numbering and naming convention given in Figure 1.

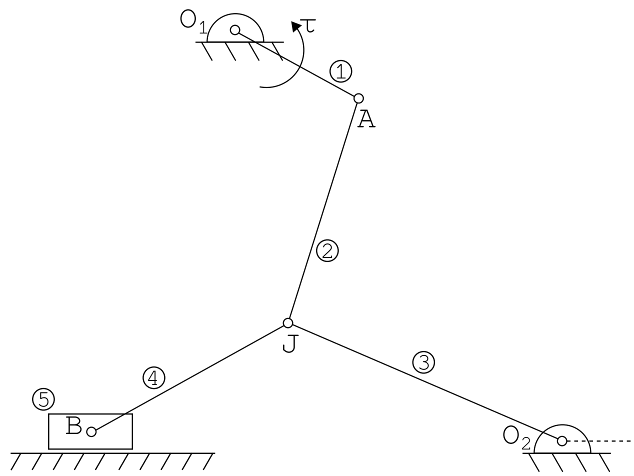
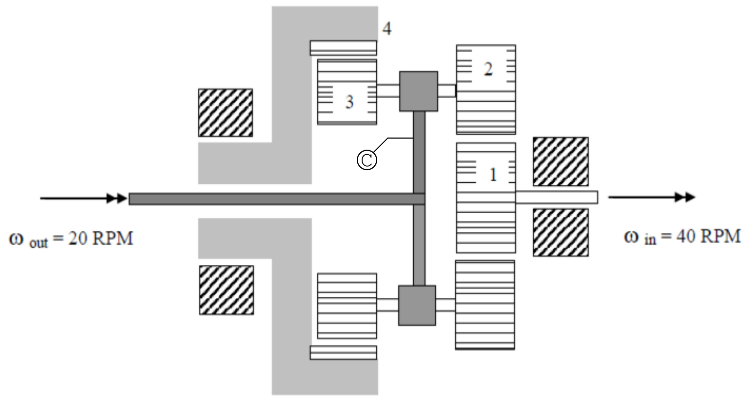


Figure 1: `Toggle Linkage`

(a) [17 points] Draw the free body diagram of each rigid body, marking all the external forces and moments as well as the inertial forces and moments. (Hint: To simplify the free-body diagrams at the multiple link junction, consider the point J as an auxiliary rigid body with no mass.)

(b) [23 points] *Forward dynamics* is the problem of finding the link accelerations ${}^0\alpha^k,\ k = 1, \ldots, 4$ and ${}^0a^B$, given the input torque, τ. Derive the Newton-Euler equations for forward dynamics in vector form. There is **no need** to write out the matrix form of the equations. Clearly state what the unknowns are and how many of them there are. Count the number of equations you have derived to solve for these unknowns.

3. [25 points] The ring gear in the epicyclic geartrain shown below is *not* fixed to the ground. The tooth numbers are $N_1 = 27$, $N_2 = 36$, $N_3 = 24$, $N_4 = 87$. Find the angular velocity of the ring gear. (Hint: Gears 2 and 3 are parts of the same rigid body.)



4. [15 points] A CEP (critical extreme position) cam is being designed to provide the following motion to a translating follower

Follower Displacement		Cam Rotation [rad]
Rise	0 cm to 20 cm	0 rad to $\frac{2\pi}{3}$ rad
Fall	20 cm to 0 cm	$\frac{2\pi}{3}$ rad to $\frac{4\pi}{3}$ rad
Dwell	at 0 cm	$\frac{4\pi}{3}$ rad to 2π rad

Find a matrix representations for the coefficients of a **single** polynomial displacement function for the rise and fall portion of the follower. Clearly state your boundary conditions at the beginning of your solution.