ME 380

Kinematics and Machine Dynamics
Final Exam

1. [30 points] Consider the Whitworth quick-return mechanism shown in Figure 1. The lengths of links 1, 2, and 4 are given by l_1, l_2, and l_4. The position of link 3 along link 2 is given by s_3 and the horizontal position of link 5 is given by s_5.

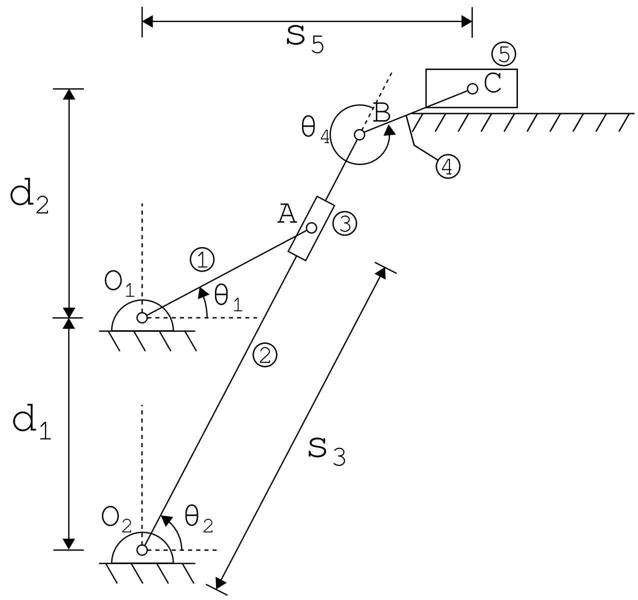


Figure 1: `Whitworth quick-return mechanism`

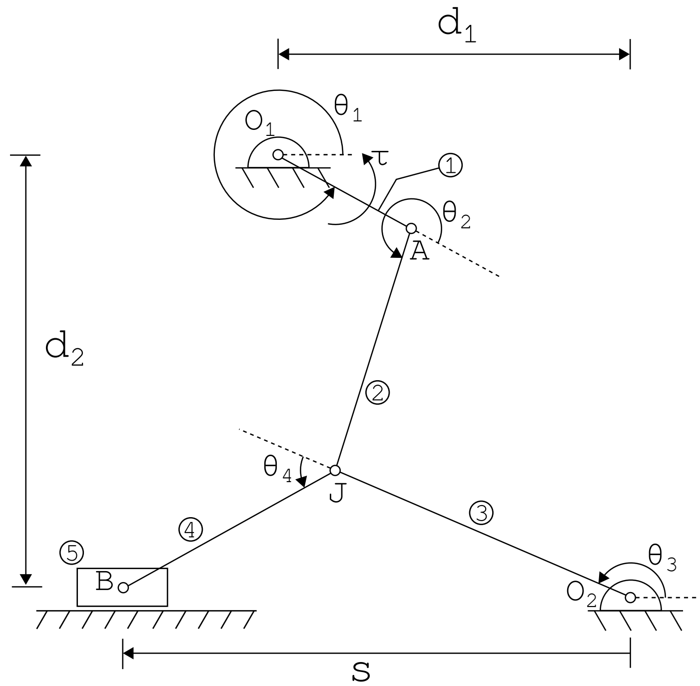
   (a) [10 points] Given θ_1, write down the loop equations from which one can solve for θ_2, s_3, θ_4, and s_5.

   (b) [10 points] Given ${}^0\omega^1 = \dot{\theta}_1 \hat{e}_3$, write down the equations to find ${}^0\omega^2$, $\dot{s}_3$, ${}^0\omega^4$, and $\dot{s}_5$ in matrix form.

   (c) [10 points] Given ${}^0\alpha^1 = \ddot{\theta}_1 \hat{e}_3$, write down the equations to find ${}^0\alpha^2$, $\ddot{s}_3$, ${}^0\alpha^4$, and $\ddot{s}_5$ (You can leave the equations in vector form, i.e., no need to write out the matrix form of the equations).

2. [35 points] Consider the **toggle linkage** shown in the figure below. Links 1 through 4 have lengths $l_1, \ldots, l_4$, and their masses are uniformly distributed. Gravity acts in the downward direction within the plane of operation. Assume that the center of mass of link 5 is at the point B and that all the links have unit masses. The moment of inertia of the all links equals 10^{-3}. The values of the link lengths are defined in the table below.

Quantity	Length [units]
l_1	1.3
l_2	2.5
l_3	3.2
l_4	2.4
d_1	3.5
d_2	4.2



(a) [10 points] Draw the free-body diagram of each rigid body, marking all the external forces and moments as well as the inertial forces and moments. (Hint: To simplify the free-body diagrams at the multiple link junction, consider the point J as an auxiliary rigid body with no mass.)

(b) [10 points] Derive the matrix form of the equations for static force balance.

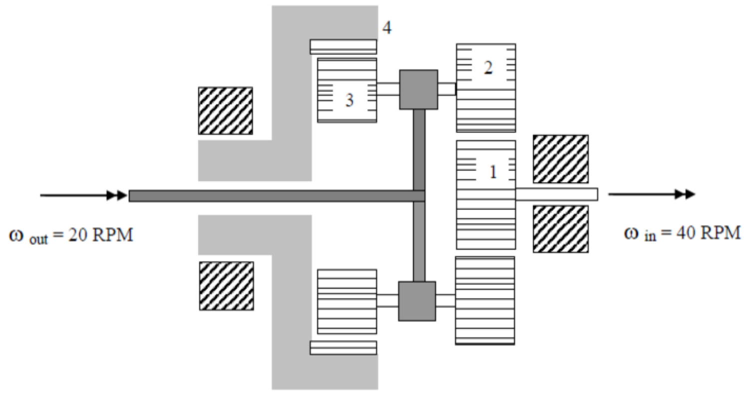
(c) [10 points] Derive the matrix form of the Newton-Euler equations for forward dynamics. (Forward dynamics is the problem of finding the link accelerations ${}^0\alpha^k$, $k = 1, \ldots, 4$ and ${}^0a^B$, given the input torque, τ.)

(d) [5 points] Assume that the kinematics problem has been solved as follows

$(\theta_1, \dot{\theta}_1, \ddot{\theta}_1)$	$\left(-\frac{\pi}{6}, -\frac{1}{2}, 0\right)$
$(\theta_2, \dot{\theta}_2, \ddot{\theta}_2)$	$(-1.298, 0.543, 0.179)$
$(\theta_3, \dot{\theta}_3, \ddot{\theta}_3)$	$(2.781, 0.197, -0.033)$
$(\theta_4, \dot{\theta}_4, \ddot{\theta}_4)$	$(0.850, -0.475, 0.101)$
$(s, \dot{s}, \ddot{s})$	$(-5.113, -0.536, -0.394)$

If at this instant $F_{01} = \begin{bmatrix} 3.282 & 28.791 \end{bmatrix}^\top$ and $F_{03} = \begin{bmatrix} -3.081 & 6.094 \end{bmatrix}^\top$, find F_{05}.

3. [25 points] The ring gear in the epicyclic geartrain shown below is *not* fixed to the ground. The tooth numbers are $N_1 = 17$, $N_2 = 28$, $N_3 = 15$, $N_4 = 60$. Find the angular velocity of the ring gear.



4. [15 points] A CEP (critical extreme position) cam is being designed to provide the following motion to a translating follower

Follower Displacement		Cam Rotation [rad]
Rise	0 cm to 5 cm	0 rad to $\frac{\pi}{2}$ rad
Fall	5 cm to 0 cm	$\frac{\pi}{2}$ rad to π rad
Dwell	at 0 cm	π rad to 2π rad

Find a matrix representations for the coefficients of a **single** polynomial displacement function for the rise and fall portion of the follower. Clearly state your boundary conditions at the beginning of your solution.