ME 380: Kinematics and Machine Dynamics
Static and Dynamic Analysis

1 How to use this assignment

In this assignment, students will learn how to perform static and dynamic analysis of mechanisms. This will involve first performing kinematic analysis and following this up with force/torque balance analysis.

- The instructor will provide questions and provide intermittent help.
- This activity provides the opportunity for the students to learn how to perform velocity-level analysis by means of several examples. They will form the relevant kinematics equations and solve them numerically.
- Students will spend approximately 30 minutes on this activity.
- This activity will be submitted and graded.

2 Learning objectives

After completing this assignment, students will be able to

- Students will be able to draw free-body diagrams.
- They will derive static and dynamic balance equations
- They will solve the static and dynamic balance equations to find the required forces/torques or accelerations.

3 Assignment completion instructions

1. Planning
   - What will the instructor need before the activity?
   - What will the student need before the activity?
     - Pencil and paper
     - Calculator
2. Source material(s) for the activity
   - Lecture notes
   - Resources placed on Piazza.
3. Introduction and warm-up practice activity for the students

- Introduction and instructions
  - Perform appropriate level kinematic analysis.
  - Draw the free body diagrams for each massive object.
  - Derive the static or dynamic balance equations.
  - Solve these equations for forward or inverse dynamics.
- Warm-up and practice
  - Brainstorm
  - Think, pair, share:
    * Recall how to draw free body diagrams.
    * Use equivalence/replacement to summarize the forces/torques on each rigid body.
    * Form a linear equation $Ax = b$ where and solve for the unknowns in the vector x.

4. Main activity
   - Students can submit a soft-copy via email or hand-in a hard-copy per group.
   - This activity graded.
5. Wrap-up / reflection
   - Peer review
     - Have a neighboring group review your work.
     - Neighboring group should provide opinion about whether or not the solution and its development are correct.
     - After the peer review is done, the owner group tries to incorporate the suggestions and improve the solution.
   - 3-2-1 Countdown
   - One-minute paper

4 Conclusion and moving forward

This activity is aimed to teach the students how to draw free body diagrams and perform static and dynamic force analsysis after an appropriate level of kinematic analysis has been performed.

5 Grading criteria

- Preparation
- Critical thinking
- Technical expertise
- Adherence to assignment guidelines
- Timeliness

5.1 Peer reviewer notes

1. Check if position- and velocity-level equations are correct.
2. Check if the vector form of the acceleration-level equations are correct.
3. Check if the expressions of the acceleration-level equations in the form $Ax = b$ is correct.

5.2 Preparation and content

Students can prepare for this assignment by reviewing the lecture notes. Specifically, students should review ...

5.3 Creativity and communication

Students should be able to explain their thinking process to a fellow student in order to make sure that they have learnt the material.

5.4 Critical thinking

Students should think about the reaction forces that are being exerted by interconnected links on each other.

5.5 Engagement with topic

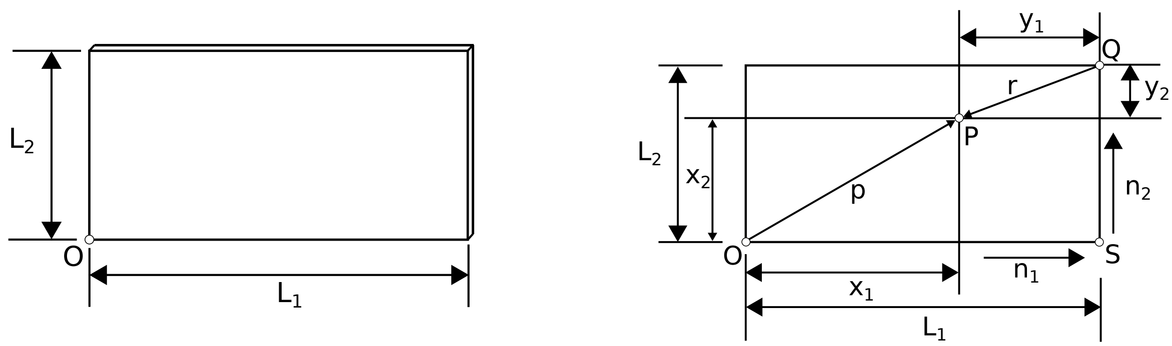
Every student should be able to and attempt to explain how they think about the problem to a fellow student.

5.6 Technical expertise

Calculus 1, Linear algebra

6 Questions

1. [25 points] The left figure below designates a thin, uniform, rectangular plate B of mass m. When B is modeled as matter distributed over a rectangular surface R, then we can represent it as the figure on the right.



   (a) Assuming that the material of the plate is uniformly distributed, compute the density (mass per unit of area), ρ, of the rectangular surface R.

   (b) [5 points] The position vector, p, from point O to a generic point P of R can be expressed as

   $$p = x_1 n_1 + x_2 n_2,$$

   as shown in the figure on the right. Use this with the definition of the inertia vector to find the inertia vector of R with respect to O for n_1.

   (c) [5 points] Assuming that the vectors n_1, n_2, and n_3 form a right-handed coordinate system, compute the inertia vector of R with respect to point Q for n_3.

   (d) [10 points] Compute the inertia matrix of R with respect to point O and R^*, the center of mass of plate (same as its geometric center), for the unit vectors n_1, n_2, and n_3. That is, compute $I^{R/O}$ and I^{R/R^*}.

   (e) [5 points] Use the appropriate parallel-axis theorem to verify your results of part (d). That is, use the parallel-axis theorem to find the inertia matrix of R with respect to point O assuming knowledge of the inertia matrix of R with respect to the center of mass R^* and then compare your result with part (d).

2. [25 points] For the planar mechanism shown in Figure 1, draw the free body diagram for each link. Write down equations of motion for each link. Identify number of equations and all the unknowns. Put equations in a matrix form.

   Assume uniform links with center of mass located at the link centers. Gravity is out of plane. Links have masses m_i and inertias J_i, $i = 1, \ldots, 5$ about their centers of mass, in the direction perpendicular to the plane. Assume that kinematic analysis has already been performed and all accelerations are known. No friction exists at the joints. (Hint: To simplify free body diagrams at the multiple link junction, consider point J as an auxiliary rigid body with no mass.)

   Refer to Figure 2 for questions 2, 3, and 4. The link lengths are given by $l_0 = 0.25\,\mathrm{m}$, $l_1 = l_4 = 0.3\,\mathrm{m}$, $l_2 = l_3 = 0.35\,\mathrm{m}$. Each link is made of homogeneous material, so assume that the center of mass of each link is at its geometric center. The inertial constants

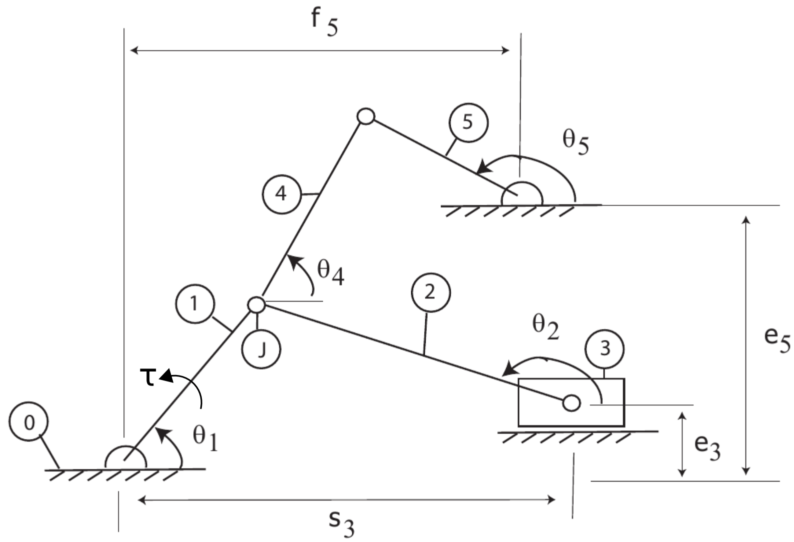


Figure 1: Planar Mechanism

are given by $m_1 = m_4 = 0.15\,\mathrm{kg}$, $m_2 = m_3 = 0.1\,\mathrm{kg}$ and $I_1 = I_4 = 1.5 \times 10^{-3}\,\mathrm{kg}\cdot\mathrm{m}^2$, $I_2 = I_3 = 2 \times 10^{-3}\,\mathrm{kg}\cdot\mathrm{m}^2$. Gravity acts downwards within the plane.

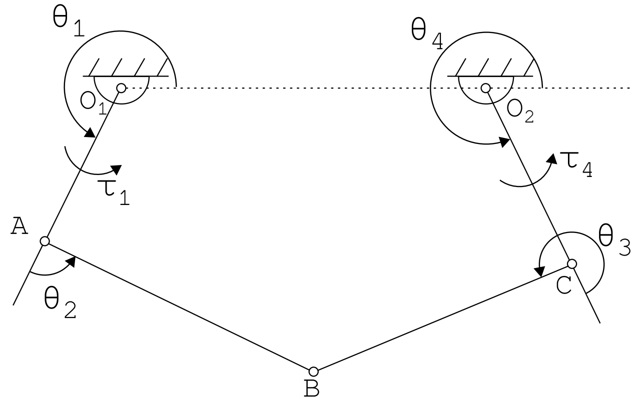


Figure 2: `Pantograph`

3. [25 points] Perform position-level kinematic analysis for the pantograph depicted in Figure 2. Follow this analysis up by performing a static-force analysis. Find the values of the motor torque $\tau = \begin{bmatrix}\tau_1 & \tau_4\end{bmatrix}^\top$ necessary to "cancel the gravity" (i.e. keep the mechanism stationary) when $\begin{bmatrix}\theta_1 & \theta_4\end{bmatrix}^\top = \begin{bmatrix}240^\circ & 310^\circ\end{bmatrix}^\top$. (Notice that you first need to find what θ_2 and θ_3 correspond to these values of θ_1 and θ_4.)

4. [25 points] Complete the kinematic analysis of the pantograph by perfoming a velocity- and acceleration-level analysis. Given that $\begin{bmatrix}\theta_1 & \theta_4\end{bmatrix}^\top = \begin{bmatrix}240^\circ & 310^\circ\end{bmatrix}^\top$ and $\begin{bmatrix}\dot\theta_1 & \dot\theta_4\end{bmatrix}^\top = \begin{bmatrix}-15 & 5\end{bmatrix}^\top \frac{\mathrm{rad}}{\mathrm{s}}$, at an instant in time, find $\begin{bmatrix}\theta_2 & \theta_3\end{bmatrix}^\top$ and $\begin{bmatrix}\dot\theta_2 & \dot\theta_3\end{bmatrix}^\top$. Additionally, find the angular acceleration of all links at this instant if $\tau = \begin{bmatrix}\tau_1 & \tau_4\end{bmatrix}^\top = \begin{bmatrix}1 & -1\end{bmatrix}^\top$ Nm.