6 Questions

Let k denote the unit vector that points perpendicularly out of the page.

1. [20 points] Referring to Figure 1, let $l_0 = 30$, $l_1 = 15$, $l_2 = 35$, $l_3 = 20$, and $\theta_1 = \frac{\pi}{3}$. Calculate θ_2, θ_3, ${}^0v^C$, ${}^0\omega^2 = \omega_2 k$, and ${}^0\omega^3 = \omega_3 k$ for ${}^0\omega^1 = \omega_1 k$ with $\omega_1 = 20\,{}^{\text{rad}}/_{\text{s}}$.

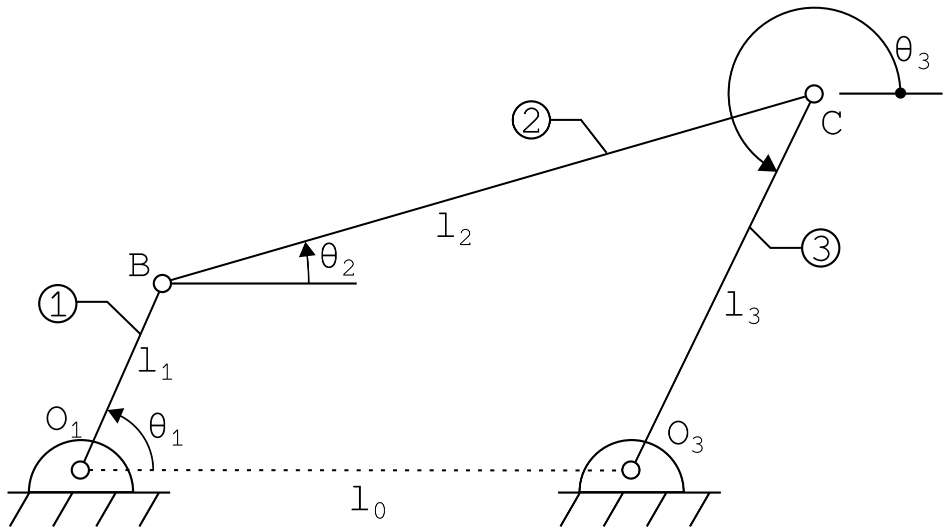


Figure 1: `Four-bar linkage`

2. [20 points] Referring to Figure 2, let $l_1 = 35$, $l_0 = 50$, $\theta_1 = {}^{\pi}/_{10}$ and ${}^0\omega^1 = \omega_1 k$ with $\omega_1 = 7.5\,{}^{\text{rad}}/_{\text{s}}$. Compute the following quantities: l_2, θ_2, ${}^0v^B$, and ${}^0\omega^2 = \omega_2 k$.

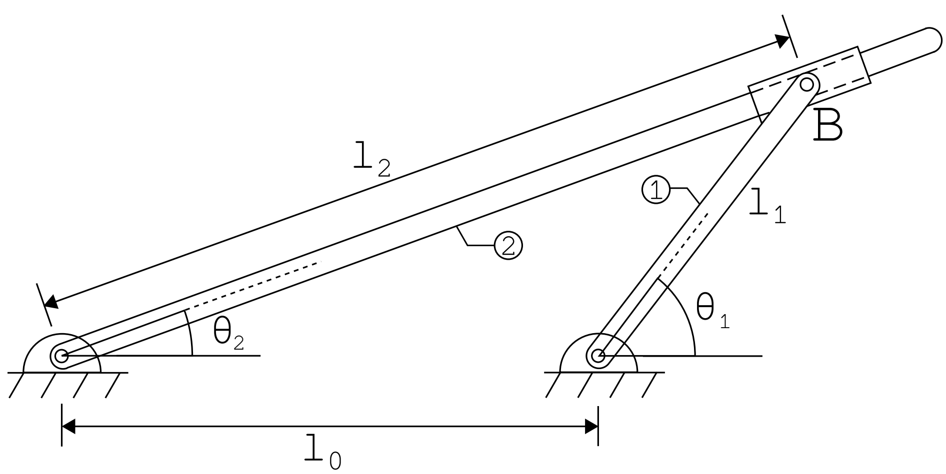


Figure 2: `Sliding contact linkage`

$l_1 = 1$
$l_2 = 7/3$
$l_3 = 4/3$

10/3

3. [15 points] Plot ${}^{\omega_3}/_{\omega_1}$ versus θ_1 curve for the linkage depicted in Figure 1 if ${}^{l_0}/_{l_1} = 2$, ${}^{l_2}/_{l_1} = {}^7/_3$, and ${}^{l_3}/_{l_1} = {}^4/_3$.

4/3

1 7/3

1

4/3

4. [20 points] Perform position, velocity, and acceleration analysis for the mechanism depicted in Figure 3 using analytical methods. Assume ${}^0\omega^1 = \omega_1 k$, ${}^0\alpha^1 = \alpha_1 k$ with $\omega_1 = 5\,{}^{\text{rad}}/_{\text{s}}$ and $\alpha_1 = 2\,{}^{\text{rad}}/_{\text{s}^2}$.

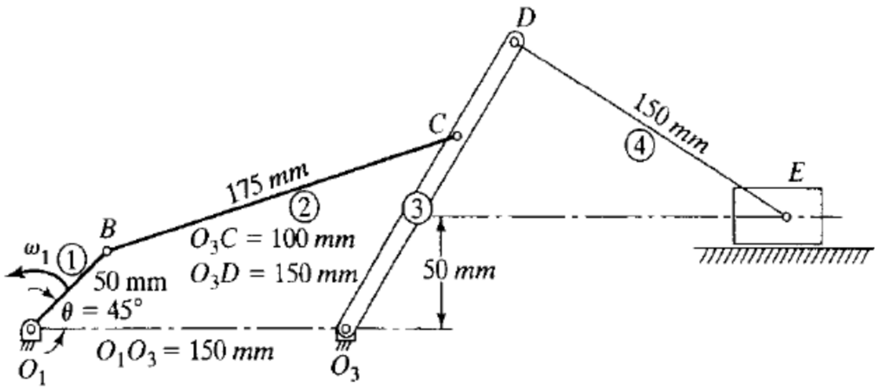


Figure 3: `Fourbar with slider`

5. [20 points] Perform position, velocity, and acceleration analysis for the mechanism shown in Figure 4 using analytical methods. Take $\theta_1 = 20°$, ${}^0\omega^1 = \omega_1 k$, ${}^0\alpha^1 = \alpha_1 k$ with $\omega_1 = {}^1/_2\,{}^{\text{rad}}/_{\text{s}}$, and $\alpha_1 = 1\,{}^{\text{rad}}/_{\text{s}^2}$ (ignore the value of ω_1 that is provided within the figure).

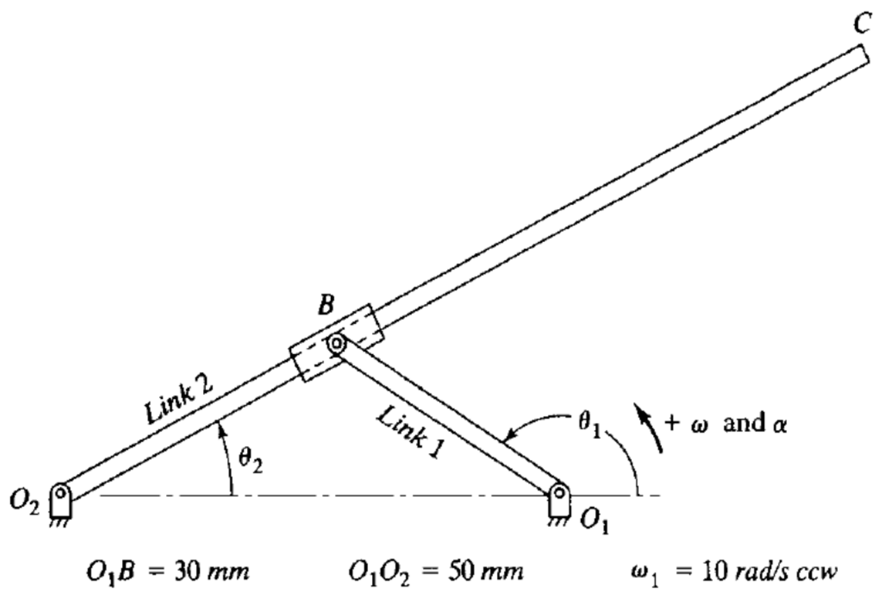
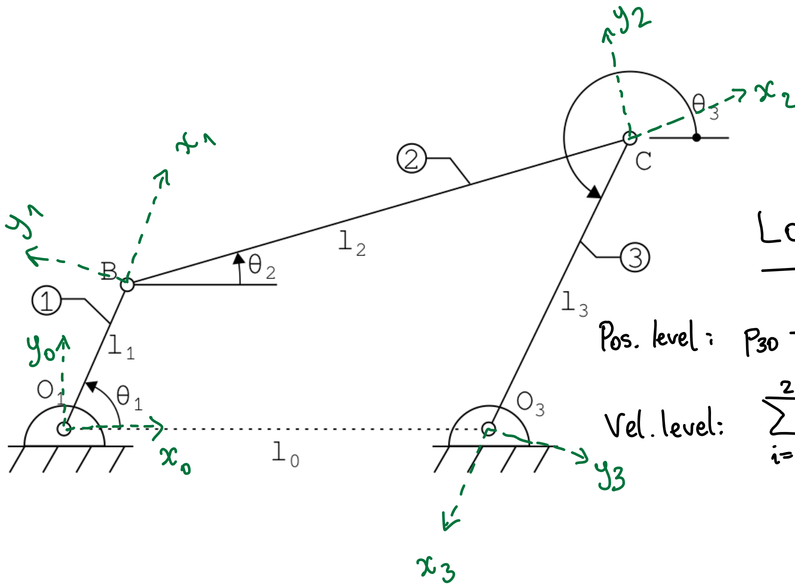


Figure 4

1



Loop equation

Pos. level: $p_{30} + \sum_{i=0}^{2} p_{i,i+1} = 0$ (1a)

Vel. level: $\sum_{i=0}^{2} {}^0\omega^{i+1} \times p_{i,i+1} = 0.$ (1b)

Expressed in Σ_0, the position and velocity-level equations are given by

$$\left.\begin{aligned} l_1\cos(\theta_1) + l_2\cos(\theta_2) + l_3\cos(\theta_3) - l_0 &= 0, \\ l_1\sin(\theta_1) + l_2\sin(\theta_2) + l_3\sin(\theta_3) &= 0. \end{aligned}\right] \text{Pos. level}$$

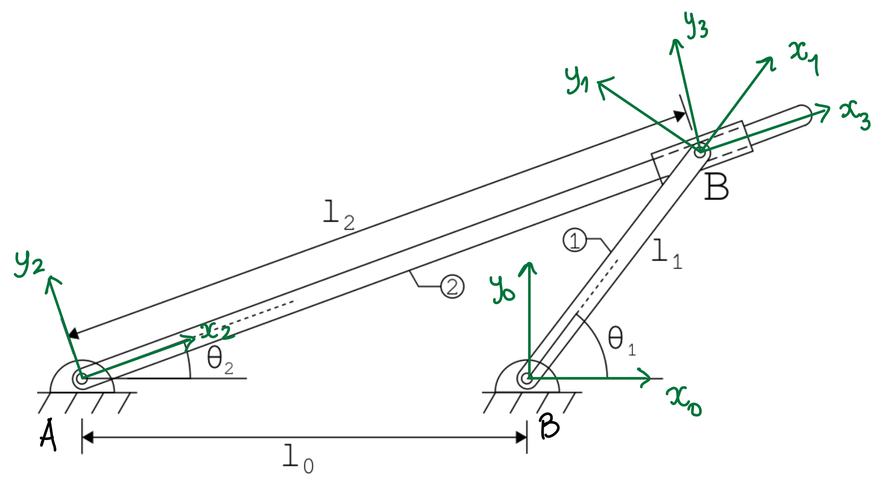
$$\left.\begin{aligned} -l_1\sin(\theta_1)\dot{\theta}_1 - l_2\sin(\theta_2)\dot{\theta}_2 - l_3\sin(\theta_3)\dot{\theta}_3 &= 0, \\ l_1\cos(\theta_1)\dot{\theta}_1 + l_2\cos(\theta_2)\dot{\theta}_2 + l_3\cos(\theta_3)\dot{\theta}_3 &= 0. \end{aligned}\right] \text{Vel. level}$$

Solving the equations for $\theta_2, \theta_3, \dot{\theta}_2$ and $\dot{\theta}_3$ yields

$$\theta_2 = 0.077 \text{ rad} = 4.433^\circ,$$
$$\theta_3 = -2.239 \text{ rad} = 4.044 \text{ rad} = -128.30^\circ,$$
$$\dot{\theta}_2 = 1.684 \text{ rad/s},$$
$$\dot{\theta}_3 = 16.843 \text{ rad/s}.$$

$${}^0v^C = \sum_{i=0}^{1} {}^0\omega^{i+1} \times p_{i,i+1} = -264.36\, x_0 + 208.77\, y_0.$$

2



Loop eqns.

Position-level

$$p_{01} + p_{12} + p_{20} = 0$$

$$\Sigma_0 \begin{cases} l_1 \cos(\theta_1) - l_2 \cos(\theta_2) - l_0 = 0 \\ l_1 \sin(\theta_1) - l_2 \sin(\theta_2) = 0 \end{cases}$$

$$l_2 = 83.986$$

$$\theta_2 = 0{,}129 \text{ rad} = 7.399^\circ$$

Velocity-level

$${}^0\omega^1 \times p_{01} + \underbrace{\frac{{}^2 d}{dt} p_{12}}_{= -{}^2v^B} + {}^0\omega^2 \times p_{12} = 0$$

$$-l_1 \sin(\theta_1)\dot{\theta}_1 - \dot{l}_2 \cos(\theta_2) + l_2 \sin(\theta_2)\dot{\theta}_2 = 0$$

$$l_1 \cos(\theta_1)\dot{\theta}_1 - \dot{l}_2 \sin(\theta_2) - l_2 \cos(\theta_2)\dot{\theta}_2 = 0$$

$$\dot{l}_2 = -48.292$$

$$\dot{\theta}_2 = 3.072 \text{ rad/s}$$

$$\Downarrow$$

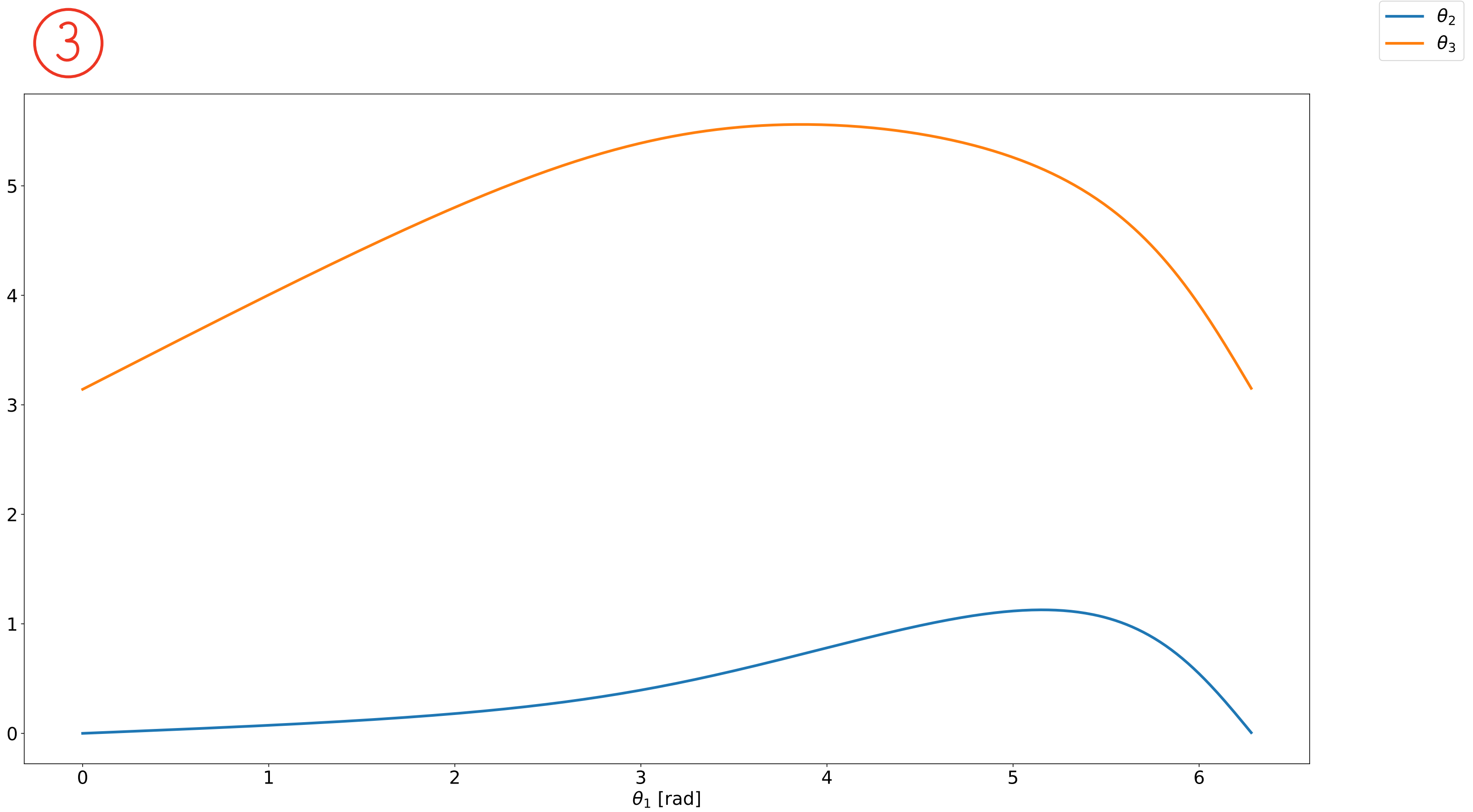
$${}^0\omega^2 = \dot{\theta}_2 z_2 = 3.072 z_2 .$$

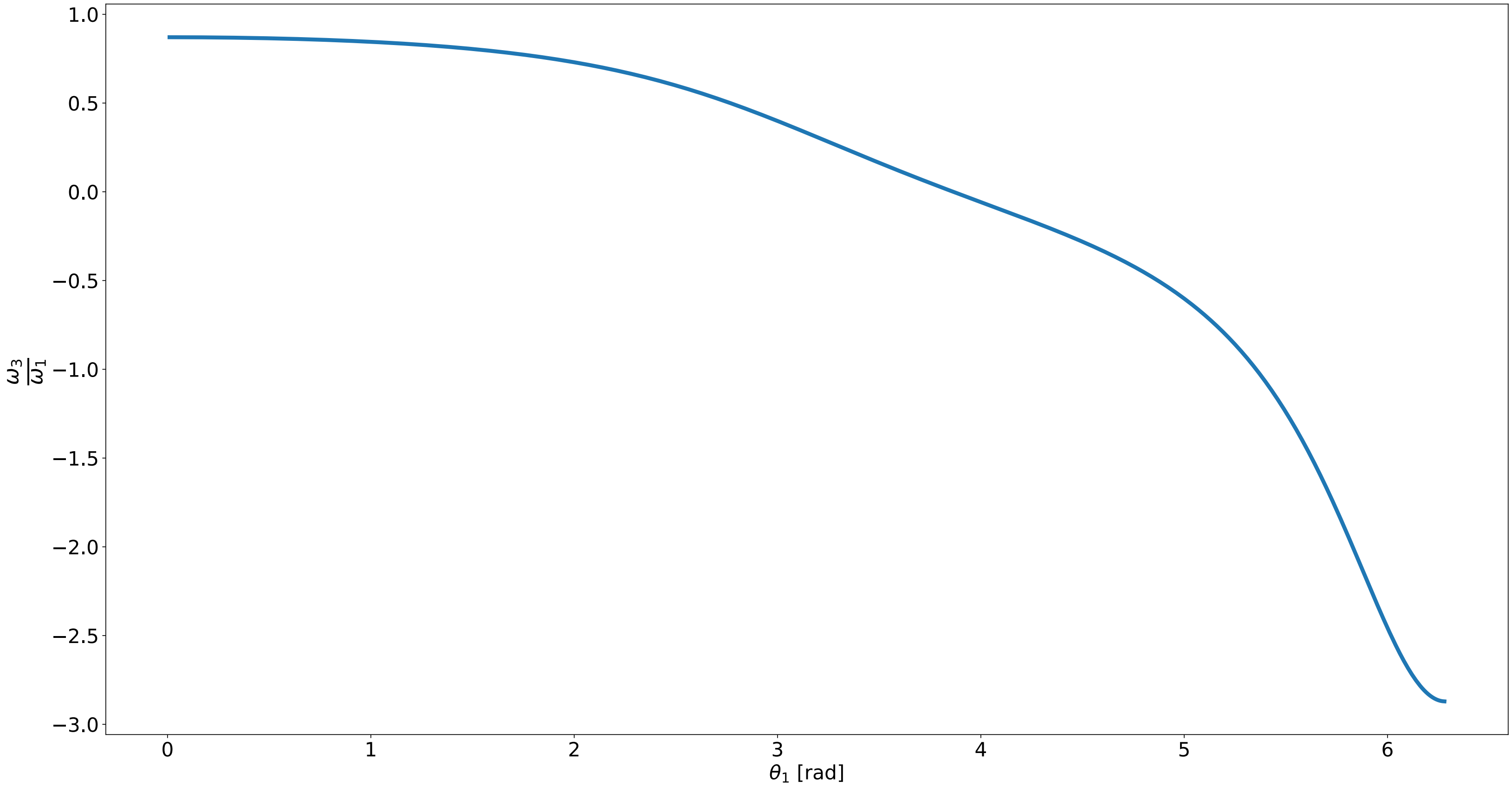
$${}^0v^B = \frac{{}^0 d}{dt}(-p_{12}) = \frac{{}^2 d}{dt}(-p_{12}) + {}^0\omega^2 \times (-p_{12}) = {}^2v^B + \omega_2 z_2 \times (l_2 x_2)$$

$$= \dot{l}_2 x_2 + l_2 \omega_2 y_2 = -48.292 x_2 + 258.0197 y_2$$

In Σ_0, $({}^0v^B)^0 = -81.117 x_0 + 249.652 y_0$, or in Σ_1, $({}^0v^B)^1 = l_1 \omega_1 y_1 = 262.5 y_1$.

③





1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
$\frac{\omega_3}{\omega_1}$
0
1
2
3
4
5
6
θ_1 [rad]

4 Define the following quantities

$$p_0 = \overrightarrow{O_3O_1}, \quad p_1 = \overrightarrow{O_1B}, \quad p_2 = \overrightarrow{BC}, \quad p_3 = \overrightarrow{CO_3}, \quad p_4 = \overrightarrow{O_3D},$$
$$p_5 = \overrightarrow{O_3D}, \quad p_6 = \overrightarrow{O_3E}.$$

There are two position-level loop equations:

$$\begin{aligned} p_0 + p_1 + p_2 + p_3 &= 0, \\ p_4 + p_5 - p_6 &= 0. \end{aligned} \tag{1}$$

Depending on how angles θ_2, θ_3 and θ_4 are defined, the expressions of these vector loop equations in the world frame change. The following uses absolute angles, for which we have the relations

$$\begin{aligned} x_i &= \cos(\theta_i)\, x_0 + \sin(\theta_i)\, y_0, \\ y_i &= -\sin(\theta_i)\, x_0 + \cos(\theta_i)\, y_0, \end{aligned} \qquad i = 1, \dots, 4.$$

$$x_5 = x_0, \quad y_5 = y_0.$$

In frame Σ_0, equations (1) become

$$\begin{aligned} l_1 \cos(\theta_1) + l_2 \cos(\theta_2) - l_3 \cos(\theta_3) - l_0 &= 0, \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_2) - l_3 \sin(\theta_3) &= 0, \end{aligned}$$

$$\begin{aligned} l_3' \cos(\theta_3) - l_4 \cos(\theta_4) - d_5 &= 0, \\ l_3' \sin(\theta_3) - l_4 \sin(\theta_4) - a_0 &= 0, \end{aligned}$$

where $\{\ell_i\}_{i=0}^{4}$ are the magnitudes of the vectors p_0, p_1, p_2, p_3, and p_5. ℓ_3' is the length of p_4 and a_0 is the vertical component of the vector p_6. Solving these equations, we get

θ_2	θ_3	θ_4	d_5
16.353°	57.808°	149.141°	208.68

Differentiating equations (1) w.r.t. time in frame 0, we get

$$\begin{aligned} \sum_{i=1}^{3} {}^0\omega^i \times p_i &= 0, \\ \sum_{i=4}^{5} {}^0\omega^i \times p_i - \dot{d}_5 x_0 &= 0. \end{aligned} \tag{2}$$

Writing these out in the Σ_0 frame and putting into matrix form, we get $Jv = 0$, where $v = (\dot{\theta}_1 \; \dot{\theta}_2 \; \dot{\theta}_3 \; \dot{\theta}_4 \; \dot{d}_5)^T$ and

$$J = \begin{bmatrix} -\ell_1 \sin(\theta_1) & -\ell_2 \sin(\theta_2) & \ell_3 \sin(\theta_3) & 0 & 0 \\ \ell_1 \cos(\theta_1) & \ell_2 \cos(\theta_2) & -\ell_3 \cos(\theta_3) & 0 & 0 \\ 0 & 0 & -\ell_3' \sin(\theta_3) & \ell_4 \sin(\theta_4) & -1 \\ 0 & 0 & \ell_3' \cos(\theta_3) & -\ell_4 \cos(\theta_4) & 0 \end{bmatrix}. \tag{3}$$

Solving $Jv = 0$ at the current configuration for $\dot{\theta}_1 = \omega_1 = 5$, we obtain

$\dot{\theta}_2$	$\dot{\theta}_3$	$\dot{\theta}_4$	$\dot{d}_5$
−0.478	1.810	−1.120	−316.26

Differentiating equations (2) w.r.t. time in Σ_0, we obtain

$$\sum_{i=1}^{3} {}^0\alpha^i \times p_i + {}^0\omega^i \times ({}^0\omega^i \times p_i) = \sum_{i=1}^{2} \left(l_i \ddot{\theta}_i y_i - l_i \dot{\theta}_i^2 x_i \right) - l_3 \ddot{\theta}_3 y_3 + l_3 \dot{\theta}_3^2 x_3 = 0,$$

$$\sum_{i=4}^{5} {}^0\alpha^i \times p_i + {}^0\omega^i \times ({}^0\omega^i \times p_i) - \ddot{d}_5 x_0 = l_3' \ddot{\theta}_3 y_3 - l_3' \dot{\theta}_3^2 x_3 - l_4 \ddot{\theta}_4 y_4 + l_4 \dot{\theta}_4^2 x_4 - \ddot{d}_5 x_0 = 0.$$

Writing these out in frame Σ_0, we obtain

$$J \dot{v} + \dot{J} v = 0,$$

where J is given in equation (3) and $\dot{J}$ denotes the matrix, whose entries are the time-derivatives of the corresponding entries of J. Solving this equation for the unknowns in v yields

$\ddot{\theta}_2$	$\ddot{\theta}_3$	$\ddot{\theta}_4$	$\ddot{d}_5$
7.760	14.189	-6.329	-2712.519

5 Position-level loop equation

$$p_0 + p_1 - p_2 = 0. \qquad (1)$$

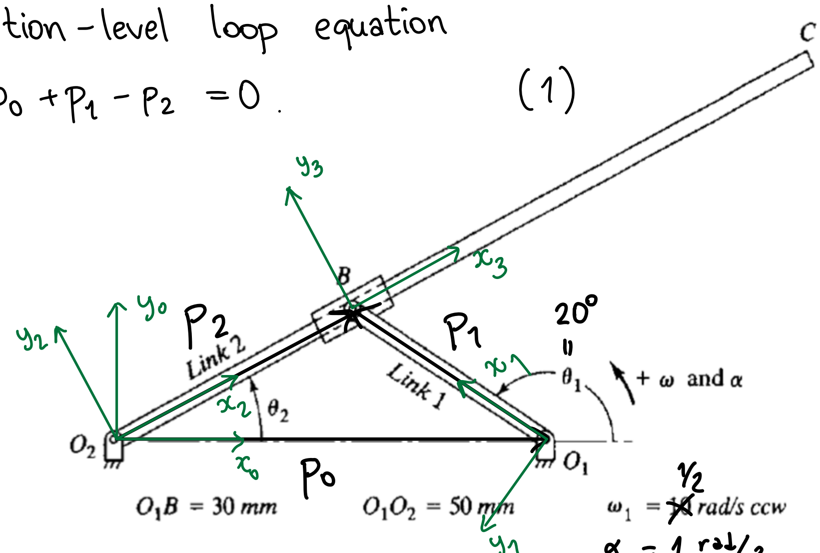


Figure 4

Writing out equation (1) in Σ_0, we obtain

$$\left.\begin{aligned} l_0 + l_1\cos(\theta_1) - d_3\cos(\theta_2) &= 0, \\ l_1\sin(\theta_1) - d_3\sin(\theta_2) &= 0. \end{aligned}\right\} \quad \boxed{\begin{aligned} \theta_2 &= 7.476° \\ d_3 &= 78.861\ mm. \end{aligned}}$$

Differentiating equation (1) w.r.t. time in frame Σ_0 yields

$$^0\omega^1 \times p_1 - {}^2v^B - {}^0\omega^2 \times p_2 = 0 \qquad (2)$$

Expressing eqn. (2) in frame Σ_0 yields

$$\left.\begin{aligned} -l_1\sin(\theta_1)\dot\theta_1 + d_3\sin(\theta_2)\dot\theta_2 - \dot d_3\sin(\theta_2) &= 0, \\ l_1\cos(\theta_1)\dot\theta_1 - d_3\cos(\theta_2)\dot\theta_2 - \dot d_3\cos(\theta_2) &= 0. \end{aligned}\right\} \quad \boxed{\begin{aligned} \dot\theta_2 &= 0.186\ \tfrac{rad}{s} \\ \dot d_3 &= -3.253\ mm/s. \end{aligned}}$$

Differentiating equation (2) w.r.t. time in frame Σ_0 yields

$$^0\alpha^1 \times p_1 + {}^0\omega^1 \times ({}^0\omega^1 \times p_1) - {}^2a^B - 2\,{}^0\omega^2 \times {}^2v^B - {}^0\alpha^2 \times p_2 - {}^0\omega^2 \times ({}^0\omega^2 \times p_2) = 0. \quad (3)$$

Expressing equation (3) in frame Σ_0 yields

$$-\ell_1 \sin(\theta_1)\ddot{\theta}_1 + d_3 \sin(\theta_2)\ddot{\theta}_2 - \ddot{d}_3 \cos(\theta_2) = \ell_1 \cos(\theta_1)\dot{\theta}_1^2 - 2\sin(\theta_2)\dot{\theta}_2 \dot{d}_3 - d_3 \cos(\theta_2)\dot{\theta}_2^2 ,$$

$$\ell_1 \cos(\theta_1)\ddot{\theta}_1 - d_3 \cos(\theta_2)\ddot{\theta}_2 - \ddot{d}_3 \sin(\theta_2) = \ell_1 \sin(\theta_1)\dot{\theta}_1^2 + 2\cos(\theta_2)\dot{\theta}_2 \dot{d}_3 - d_3 \sin(\theta_2)\dot{\theta}_2^2 .$$

Solving these equations for the unknowns yields

$$\boxed{\begin{aligned} \ddot{\theta}_2 &= 0.366 \ \mathrm{rad}/\mathrm{s}^2 , \\ \ddot{d}_3 &= -11.108 \ \mathrm{mm}/\mathrm{s}^2 . \end{aligned}}$$