ME 380: Kinematics and Machine Dynamics
Angular Velocity

1 How to use this assignment

In this assignment, students will learn how to compute the angular velocity of rigid bodies. This computation involves being able to differentiate vectors in different frames.

- The instructor will provide questions and provide intermittent help.
- This activity provides the opportunity for the students to learn how to perform velocity-level analysis by means of several examples. They will form the relevant kinematics equations and solve them numerically.
- Students will spend approximately 30 minutes on this activity.
- This activity will be submitted and graded.

2 Learning objectives

After completing this assignment, students will be able to

- Students will be able to compute angular velocities of rigid bodies moving in 3D-space.
- They will use differentiation rules to deduce properties of the angular velocity vector.

3 Assignment completion instructions

1. Planning
   - What will the instructor need before the activity?
   - What will the student need before the activity?
     - Pencil and paper
     - Calculator
2. Source material(s) for the activity
   - Lecture notes
   - Resources placed on Canvas.
3. Introduction and warm-up practice activity for the students
   - Introduction and instructions
     - Write down the loop equations in vector form.

  - Use differentiation rules to take time derivatives of these equations.
  - Express the equations in a certain coordinate frame (e.g. base frame).
- Warm-up and practice
  - Brainstorm
  - Think, pair, share:
    * Remember how to differentiation in two frames (this is the most important formula!)
    * Think about how to effectively use differentiation in two frames formula; remember, time derivative of a constant function is zero.
    * When do we use Newton-Raphson to solve equations and when can we use linear techniques?

4. Main activity
   - Students can submit a soft-copy via email or hand-in a hard-copy per group.
   - This activity graded.
5. Wrap-up / reflection
   - Peer review
     - Have a neighboring group review your work.
     - Neighboring group should provide opinion about whether or not the solution and its development are correct.
     - After the peer review is done, the owner group tries to incorporate the suggestions and improve the solution.
   - 3-2-1 Countdown
   - One-minute paper

4 Conclusion and moving forward

This activity is aimed to teach the students how to compute the angular velocity of rigid bodies and interpret the resulting vector.

5 Grading criteria

- Preparation
- Critical thinking
- Technical expertise
- Adherence to assignment guidelines
- Timeliness

5.1 Peer reviewer notes

1. Check if differentiation has been performed correctly.
2. Check if the matrix algebra is performed correctly.

5.2 Preparation and content

Students can prepare for this assignment by reviewing the lecture notes. Specifically, students should review ...

5.3 Creativity and communication

Students should be able to explain their thinking process to a fellow student in order to make sure that they have learnt the material.

5.4 Critical thinking

Students should try and visualize how the links would move if a given driver link moves in a given direction. This will help verify the results found after the mathematical manipulations.

5.5 Engagement with topic

Every student should be able to and attempt to explain how they think about the problem to a fellow student.

5.6 Technical expertise

Calculus 1, Linear algebra

6 Questions

If $a = \begin{pmatrix} a_x & a_y & a_z \end{pmatrix}$ is a 3-vector, we define the skew-symmetrix matrix $S(a)$ as

$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}.$$

1. [10 points] Verify, by direct calculation, that the operator S is linear; that is,

$$S(\alpha a + \beta b) = \alpha S(a) + \beta S(b),$$

for any vectors a and b belonging to $\mathbb{R}^3$ and scalars α and β.

2. [10 points] Verify, by direct calculation, that for any vectors a and p belonging to $\mathbb{R}^3$,

$$S(a)p = a \times p$$

where $a \times p$ denotes the vector cross product.

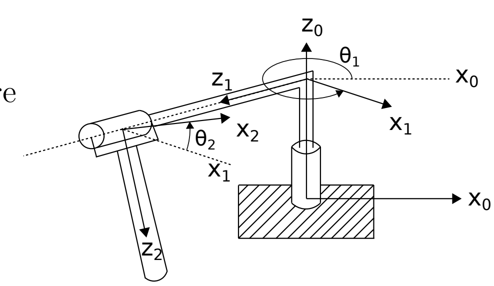
3. [10 points] Let $i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top$, $j = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^\top$, and $k = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top$. Verify, by direct calculation, that

$$\frac{\mathrm{d}}{\mathrm{d}\theta} R_{x,\theta} = R_{x,\theta} S(i), \qquad \frac{\mathrm{d}}{\mathrm{d}\theta} R_{y,\theta} = R_{y,\theta} S(j), \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}\theta} R_{z,\theta} = R_{z,\theta} S(k)$$

4. [20 points] Compute the angular velocity of both links of the Furuta pendulum, shown below, relative to the world and express them in the world frame.

Hint: Observe that the rotation matrices are given by

$$R_{i,i+1} = R_{z,\theta_{i+1}} R_{x,\pi/2}, \quad i = 0, 1.$$



5. [10 points (bonus)] Given any square matrix A, the exponential of A is a matrix defined as

$$e^A := \sum_{n=0}^{\infty} \frac{1}{n!} A^n = I + A + \frac{1}{2} A^2 + \frac{1}{3!} A^3 + \cdots.$$

Given a skew symmetrix matrix $S \in \mathfrak{so}(3)$, show that $SO(3) \ni e^S$ is a rotation matrix.

Hint: Use the fact that $e^A e^B = e^{A+B}$ provided that A and B commute, that is, $AB = BA$, and the fact that $\left| e^A \right| = e^{\operatorname{tr} A}$.