ME 380: Kinematics and Machine Dynamics
Displacement Analysis and Newton-Raphson Iteration

1 How to use this assignment

In this assignment, students will learn how to perform displacement analysis of mechanisms. They will also learn how to numerically solve the nonlinear equations arising from this analysis.

- The instructor will provide questions and provide intermittent help.
- This activity provides the opportunity for the students to learn how to perform displacement analysis by means of several examples. They will form the relevant kinematics equations and solve them numerically.
- Students will spend approximately 30 minutes on this activity.
- This activity will be submitted and graded.

2 Learning objectives

After completing this assignment, students will be able to

- Students will be able to place coordinate frames, write down position-level kinematic equations.
- They will pull these equations to a certain single reference frame.
- They will solve the equations using Newton-Raphson iteration.

3 Assignment completion instructions

1. Planning
    - What will the instructor need before the activity?
    - What will the student need before the activity?
        - Pencil and paper
        - Calculator
2. Source material(s) for the activity
    - Lecture notes
    - Resources placed on Piazza.
3. Introduction and warm-up practice activity for the students

- Introduction and instructions
  - Place coordinate frames so as to conveniently express desired vectors.
  - The vectors should form a loop; that is, they should add up to the zero vector.
  - Remember how to express vectors in various coordinate frames.
- Warm-up and practice
  - Brainstorm
  - Think, pair, share:
    * How many frames should you place for a given mechanism?
    * How do you relate two arbitrary frames to each other?
    * Recall the Taylor expansion of a function f around a point $x^{(k)}$.

4. Main activity
   - Students can submit a soft-copy via email or hand-in a hard-copy per group.
   - This activity graded.
5. Wrap-up / reflection
   - Peer review
     - Have a neighboring group review your work.
     - Neighboring group should provide opinion about whether or not the solution and its development are correct.
     - After the peer review is done, the owner group tries to incorporate the suggestions and improve the solution.
   - 3-2-1 Countdown
   - One-minute paper

4 Conclusion and moving forward

This activity is aimed to teach the students how to write down position-level kinematic equations and solve them using numerical methods.

5 Grading criteria

- Preparation
- Critical thinking
- Technical expertise
- Adherence to assignment guidelines
- Timeliness

5.1 Peer reviewer notes

1. Check if relationships between frames are correct.
2. Check if the vector loop equation indeed sums to zero.
3. Check if the resulting equations in a chosen frame are correct.

5.2 Preparation and content

Students can prepare for this assignment by reviewing the lecture notes. Specifically, students should review ...

5.3 Creativity and communication

Students should be able to explain their thinking process to a fellow student in order to make sure that they have learnt the material.

5.4 Critical thinking

Students should think about how many degrees of freedom a given mechanism has. This will tell them how many constraint equations there must be in order to solve for the unknown quantities.

5.5 Engagement with topic

Every student should be able to and attempt to explain how they think about the problem to a fellow student.

5.6 Technical expertise

Calculus 1, Linear algebra

6 Questions

1. [20 points] Referring to Figure 1, let $l_1 = 35$, $l_0 = 50$, and $\theta_2 = \frac{\pi}{12}$. Find θ_1 and l_2.

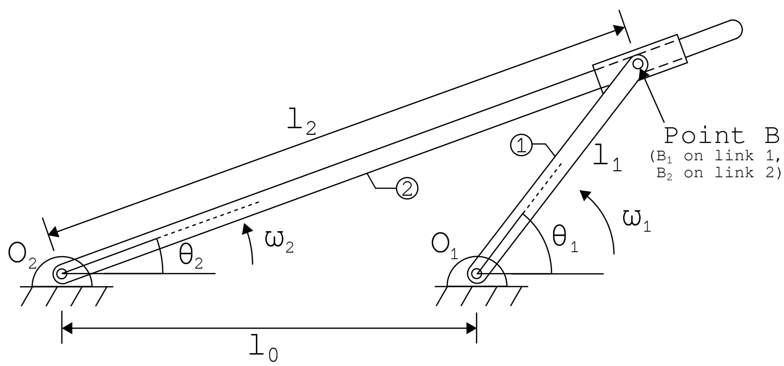

Figure 1: `Sliding contact linkage`

2. [15 points] Let r_1, r_2, and r_3 be vectors on a plane such that
$$r_1 + r_2 + r_3 = 0$$
If we additionally know that
$$\frac{r_{1y}}{r_{1x}} = 2, \quad \frac{r_{2y}}{r_{2x}} = \frac{1}{2}, \quad r_3 = \begin{bmatrix} 50 & 75 \end{bmatrix}^\top$$
Find r_1 and r_2 by using the vector cross product method.

3. [30 points] Consider the double-slider two-loop linkage illustrated in Figure 2, where $l_1 = 1.3$, $l_2 = 5.2$, $l_F = 5$, $l_{DE} = 4.5$, $l_{CD} = 1.5$, and $\theta_1 = \frac{\pi}{6}$. Find θ_2, θ_3, l_D, and l_E in two steps.

   1. Analytically derive the loop equations, $f(x) = 0$, in the base frame, where $x = \begin{bmatrix} \theta_2 & \theta_3 & l_D & l_E \end{bmatrix}^\top$.
   2. Use numerical techniques (Newton-Raphson) to produce a numerical solution.
      i. Pick your favorite coding language amongst Julia, Matlab, or Python to code the Newton-Raphson algorithm.
      ii. Use this algorithm to find an $x^\star$ such that $f(x^\star) \approx 0$ (Hint: pick your initial guess, $x^{(0)}$, wisely).
      iii. Use Julia's `NLsolve.jl` package or Python's `fsolve` command (found in `scipy.optimize`) or Matlab's `fsolve` command to find an $x^\star$ such that $f(x^\star) \approx 0$ starting from the initial guess $x^{(0)}$ you used in part (ii). Do the results you got in parts (ii) and (iii) match?

4. [20 points (bonus)] An inline slider-crank linkage has a crank length l_1 and a connecting rod length $l_2 = \frac{3}{2} l_1$ (see Figure 3). Find the connecting rod and the slider position (in terms of l_1) when the crank angle is $\theta = \frac{2\pi}{9}$ by using the vector cross product method.

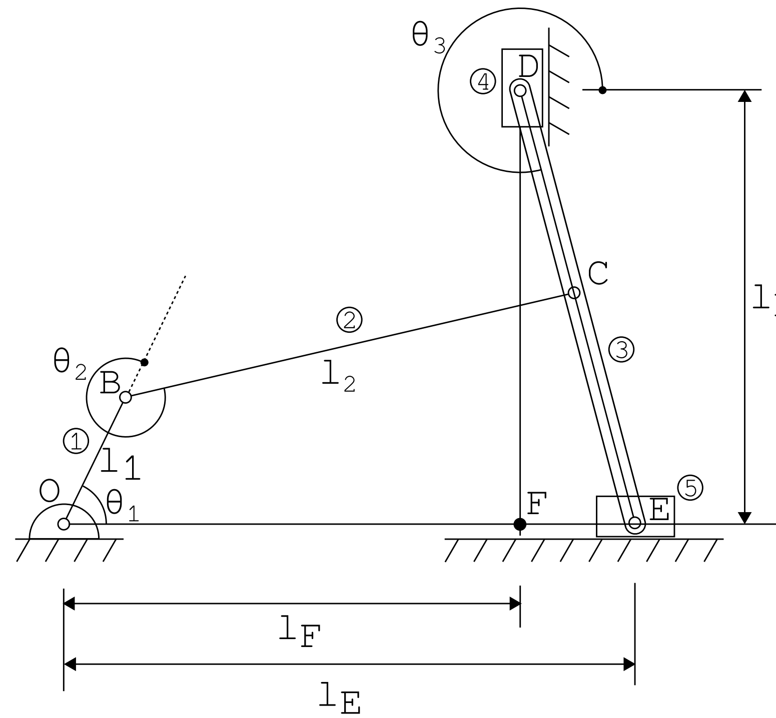

Figure 2: `Double-slider two-loop linkage`

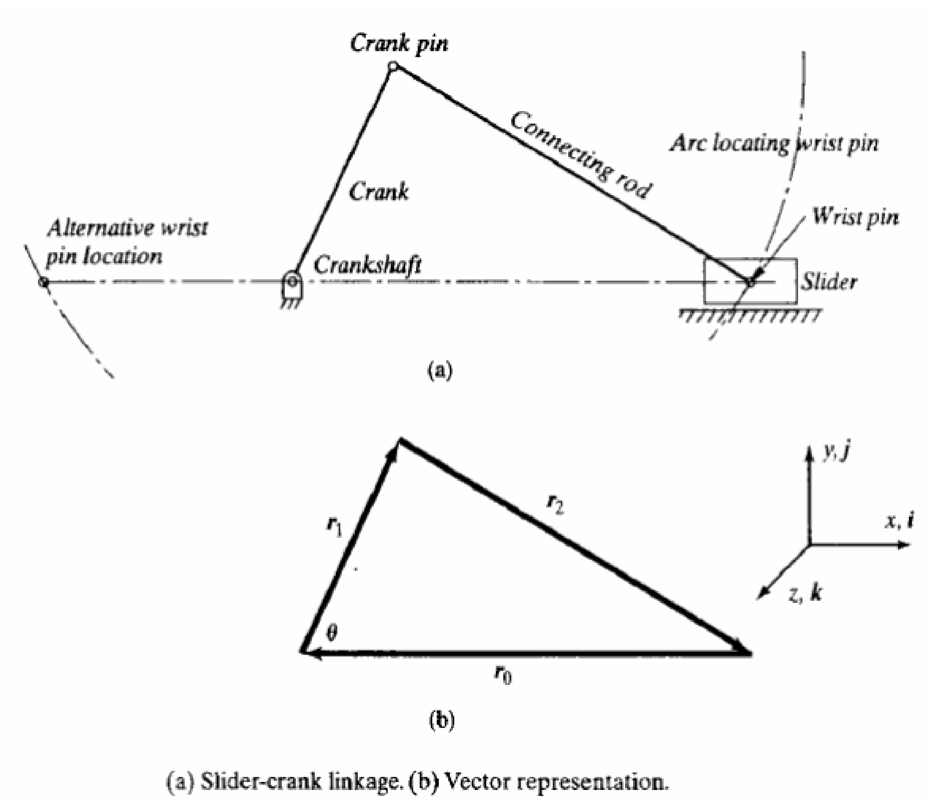

Figure 3: `Slider-Crank`